Control and Intelligent Systems, Vol. 35, No. 3, 2007

DAVISON TYPE INTEGRAL CONTROLLERS FOR TIME-DELAY PLANTS USING A SIMPLIFIED PREDICTOR

J. Wu,* T. Ishihara,** and Q.M.J. Wu***

Abstract

A suboptimal Davison type integral controller for time-delay plants is proposed which requires less on-line computation than that for the optimal controller. The proposed suboptimal controller utilizes a simplified predictor, which replaces a part of the optimal predictor with a Smith predictor. As a systematic method for designing the proposed controller the application of the loop transfer recovery (LTR) technique is discussed. For the plant input side and the plant output side, explicit representations of the sensitivity matrices achieved by enforcing the formal LTR procedure are obtained. A numerical example is presented to compare the asymptotic feedback properties with those of the optimal design.

Key Words

Time-delay plants, integral controller, Davison type controller, loop transfer recovery, state predictor, Smith predictor

1. Introduction

Integral controller design for a time-delay plant is often required in various control applications. It is well-known that an important class of the integral controllers can be obtained as a special case of the robust servosystems proposed by Davison [1]. The Davison type integral controller consists of the external feedback loop for integral compensation and the internal loop for the observer-based stabilization. Using the separation principle, Watanabe [2] proposed a design for an optimal Davison type integral controller for a time-delay plant. The optimal controller includes the optimal predictor for the extended plant, consisting of the plant and the integrators inserted to the plant output. As the optimal predictor requires on-line numerical integrations, the computational requirement for the optimal controller is relatively high. It is desirable to reduce the amount of on-line computation for practical applications.

In this paper, we propose a suboptimal Davison type integral controller, which requires less on-line computation for a time-delay plant. By a simple trick with the separation principle, the suboptimal controller is constructed by replacing a part of the optimal predictor with a Smith predictor [3].

As a systematic method for determining the design parameters of the proposed controller, we consider the application of the loop transfer recovery (LTR) technique [4, 5]. The LTR techniques for time-delay plants have been well discussed for the LQG controllers [6–8]. It is worth noting that Wu *et al.* [9] have discussed the LTR design of the optimal Davison type integral controllers for time-delay plants.

Although the simplified predictor is used, the structure of the proposed controller is still complex when compared with the standard LQG controller. For transparent discussion of the feedback properties, we use decompositions of the sensitivity matrices rather than the loop transfer function matrices. The decompositions are related to a factorization of the controller transfer function matrix. Using the decompositions, we can easily derive explicit representations of the sensitivity matrices achieved by enforcing the formal LTR procedure. The meaning of enforcing the formal LTR procedure is discussed using the explicit representations. A numerical example is presented to compare the achievable feedback properties with those of the optimal controller.

Section 2 introduces the suboptimal controller using a simplified predictor. In Section 3 the application of the LTR procedure is discussed. In Section 4 a numerical example is presented to illustrate the performance of the suboptimal controller. Concluding remarks are given in Section 5.

2. Suboptimal Controller Using a Simplified Predictor

2.1 Problem Formulation

Consider a plant with a time-delay:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t+\tau) = Cx(t) \tag{1}$$

* Canadian Dehua International Trading Ltd., 1411-409 Granville Street, Vancouver, BC, Canada, V6C 1T2; e-mail: taylor.wu@dehua.ca
** Faculty of Science and Technology, Fukushima University, 1 Kanayagawa, Fukushima, Japan, 960-1296; e-mail: ishihara@sss.fukushima-u.ac.jp
*** Department of Electrical and Computer Engineering, The University of Windsor, 401 Sunset, Windsor, Ontario, Canada; e-mail: jwu@uwindsor.ca
Recommended by Dr. Mohammed Sarhan
(paper no. 201-1758)

where $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is an output vector, $y(t) \in R^m$ is a control input and τ is a time-delay. It is assumed that $(A,\ B,\ C)$ is minimal and minimum phase. In addition, $C(sI - A)^{-1}B$ is assumed to be non-singular for almost all s.

For the time-delay plant (1), we propose a suboptimal Davison type integral controller using a simplified predictor. The controller is constructed by a simple trick with the separation principle.

2.2 Optimal State Feedback Design For Delay-Free Case

Assume that the plant is delay-free ($\tau = 0$) and that the state $x(t)$ is measurable. Consider the extended system

$$\dot{\xi}(t) = \Phi\xi(t) + \Gamma u(t), \quad \eta(t) = H\xi(t) \tag{2}$$

where $\eta(t) \in R^m$ is a state vector of the integrators and

$$\xi(t) \triangleq [x'(t)\ \ \eta'(t)]',\ \ \Phi \triangleq \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix},\ \ \Gamma \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix},\ \ H \triangleq [0\ \ I] \tag{3}$$

Define the quadratic performance index as

$$J \triangleq \int_0^\infty [\eta'(t)Q\eta(t) + u'(t)Ru(t)]dt \tag{4}$$

where $Q > 0$ and $R > 0$. Consider a non-negative definite solution P of the Riccati equation:

$$\Phi'P + P\Phi - P\Gamma R^{-1}\Gamma'P + H'QH = 0 \tag{5}$$

Then the optimal state feedback gain matrix is given by

$$F \triangleq R^{-1}\Gamma'P \tag{6}$$

Define the partition of F compatible with the vector $\xi(t)$ in (3) as

$$F = [F_x \quad F_y] \tag{7}$$

Using the state feedback matrices (8), we can construct a state feedback integral controller as

$$u_0(t) = -F_x x(t) + F_y \int_0^t [r(\sigma) - y(\sigma)]d\sigma \tag{8}$$

where $r(t)$ is a reference input vector and $y(\sigma) = Cx(\sigma)$. The structure of the optimal state feedback integral controller for a delay-free case is shown in Fig. 1.

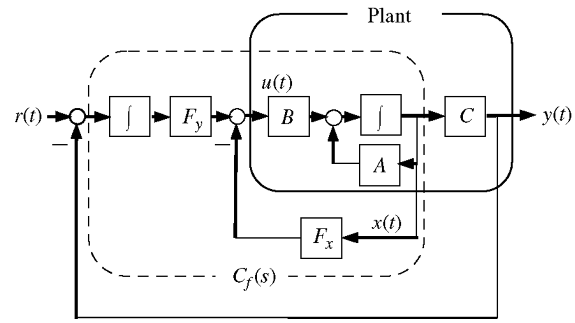

Figure 1. State feedback integral controller for delay-free plant.

2.3 Suboptimal Controller for Time-Delay Plant

A suboptimal controller for the time-delay plant is constructed by a simple trick with the separation principle. Let us regard the control system in Fig. 1 as a unity feedback control system for a fictitious plant given by the matrix C and a fictitious controller $C_f(s)$, corresponding to the area enclosed by the dotted line in Fig. 1. Note that the time-delay plant (1) can be obtained by replacing the fictitious delay-free plant, C, in Fig. 1 with the fictitious time-delay plant $e^{-\tau s}C$. Applying the well known Smith method [3] to the time-delay plant $e^{-\tau s}C$, we can easily show that the controller, $C_f(s)$, for the delay-free plant C can be used for the time-delay plant $e^{-\tau s}C$, as shown in Fig. 2, where $(1 - e^{-\tau s})C$ is the Smith predictor.

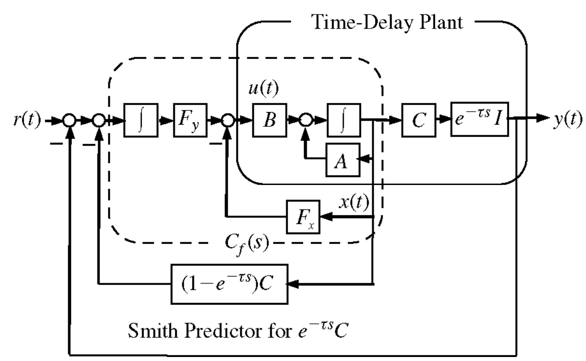

Figure 2. State feedback integral controller with the Smith predictor for $e^{-\tau s}C$.

The control input for the real time-delay plant in Fig. 2 is given by

$$\begin{aligned} u(t) &= -F_x x(t) + F_y \int_0^t [r(\sigma) - y(\sigma)]ds \\ &\quad - F_y \int_0^\tau C[x(\sigma) - x(\sigma - \tau)]d\sigma \end{aligned} \tag{9}$$

By the separation principle, the output feedback controller can be realized by replacing the state $x(t)$ by its predicted value. The prediction of $x(t)$, based on the measured output $y(\sigma)$ $(0 \le \sigma \le t)$, is obtained by cascade connection of an observer and a state predictor. The observer for (1) is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K[y(t+\tau) - C\hat{x}(t)] \tag{10}$$

where $\hat{x}(t)$ is an estimate of $x(t)$ based on the measured output up to $t+\tau$. Note that, at time t, the observer generates the state estimate $\hat{x}(t-\tau)$. The estimate is used in the state predictor to generate the prediction of the state $x(t)$, based on the measured output up to t

$$\hat{x}_\tau(t) = e^{A\tau}\hat{x}(t-\tau) + \int_{t-\tau}^{t} e^{A(t-\sigma)}Bu(\sigma)d\sigma \tag{11}$$

Now the separation principle is applied to obtain an output predictor feedback controller. From (9) and (11), the control input for the output feedback case can be defined as

$$\begin{aligned} u(t) \ = \ & -F_x\hat{x}_\tau(t) + F_y\int_0^t [r(\sigma) - y(\sigma)]\, d\sigma \\ & - F_y\int_0^t C[\hat{x}_\tau(\sigma) - \hat{x}_\tau(\sigma-\tau)]\, d\sigma \end{aligned} \tag{12}$$

The structure of the output feedback controller is shown in Fig. 3. The Laplace transform of the control input (12) is expressed as

$$u(s) = \frac{1}{s}F_y[r(s) - y(s)] - F_C(s)\hat{x}_\tau(s) \tag{13}$$

where

$$F_C(s) \triangleq \frac{1}{s}(1 - e^{-\tau s})F_yC + F_x \tag{14}$$

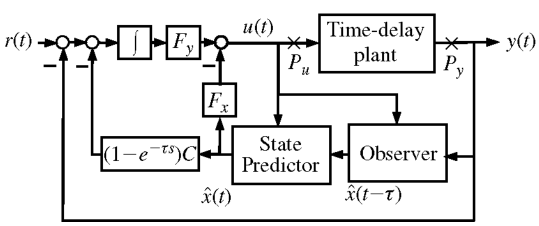

Figure 3. Output feedback integral controller for time-delay plant.

It should be noted that the output feedback controller shown in Fig. 3 is constructed by the combination of the Smith predictor and the optimal state predictor for $x(t)$. The control input (12) includes only the prediction of the state vector $x(t)$. On the other hand, the optimal predictor used by Watanabe [2] requires the numerical integration not only for the state vector $x(t)$ of the plant but also for the state vector $\eta(t)$ of the integrators. The control input of the controller using the optimal predictor is given by

$$\begin{aligned} u(t) = & -\left\{F_xe^{A\tau} + F_y\left(\int_o^\tau Ce^{A\sigma}d\sigma\right)\right\}\hat{x}(t-\tau) \\ & + F_y\int_0^t [r(\sigma) - y(\sigma)]d\sigma - F_x\left(\int_{t-\tau}^t e^{A(t-\sigma)}Bu(\sigma)d\sigma\right) \\ & - F_y\left\{\int_{t-\tau}^t\left(\int_0^{t-\tau} Ce^{A\sigma'}d\sigma'\right)Bu(\sigma)d\sigma\right\} \end{aligned} \tag{15}$$

It is obvious, from (12) and (15), that the use of the simplified predictor considerably reduces the on-line computation for the numerical integration.

3. Formal Application of LTR Procedures

To design the integral controller introduced in the previous section, the determination of the two matrices, F and K, is required. In this section, we propose the use of the LTR technique, which is a two-step method for the efficient determination of the matrices. Although the LTR techniques were originally developed for minimum phase plants, it has been widely recognized that the LTR procedures can be used for a class of non-minimum phase systems [4–11].

For the new integral controller, targets appropriate for the LTR design at the plant input side and those at the plant output side are not obvious. We clarify the feedback properties achieved by enforcing the formal LTR procedure using Riccati equations. As the structure of the controller is complex, we use the decomposition of sensitivity matrices to simplify the discussion. The explicit representations of the sensitivity matrices achieved by enforcing the formal LTR procedures are obtained. From the expressions, we can find the LTR procedures, for the new integral controller using the simplified predictor.

3.1 Plant Input Side

First, we consider the LTR design, taking notice of the feedback properties at the plant input side (the point P_u in Fig. 3). The following result is necessary to obtain the expression of the sensitivity matrix.

Proposition 1: The transfer function matrix from the measured output $y(t)$ to the input $u(t)$ in the output feedback integral controller shown in Fig. 3 can be expressed as

$$C_\tau(s) = -\overline{V}_\tau^{-1}(s)\,\overline{U}_\tau(s) \tag{16}$$

where

$$\overline{U}_\tau(s) = \frac{1}{s}F_y + F_C(s)e^{At}(sI - A + KC)^{-1}K \tag{17}$$

$$\begin{aligned} \overline{V}_\tau(s) = & I + F_C(s)(I - e^{-(sI-A)\tau})(sI-A)^{-1}B \\ & + F_C(s)e^{-(sI-A)\tau}(sI - A + KC)^{-1}B \end{aligned} \tag{18}$$

Proof: Note that

$$\begin{aligned} & \mathcal{L}\left[\int_{t-\tau}^t e^{A(t-\sigma)}Bu(\sigma)d\sigma\right] \\ & \quad = (I - e^{-(sI-A)\tau})(sI-A)^{-1}Bu(s) \end{aligned} \tag{19}$$

Using the above Laplace transform, we can write the Laplace transform of the state predictor (11) as

$$\hat{x}_\tau(s) = e^{-(sI-A)\tau}\hat{x}(s) + (I - e^{-(sI-A)\tau})(sI-A)^{-1}Bu(s) \tag{20}$$

Assume that $r(s)=0$. It follows, from (12) and (20), that

$$u(s) = -\frac{1}{s}F_y y(s) - F_C(s)[e^{-(sI-A)\tau}\hat{x}(s) + (I - e^{-(sI-A)\tau})(sI-A)^{-1}Bu(s)] \quad (21)$$

where $F_C(s)$ is defined in (14). The Laplace transform of the state estimate generated by the observer (10) can be written as

$$\hat{x}(s) = (sI - A + KC)^{-1}[Ke^{\tau s}y(s) + Bu(s)] \quad (22)$$

From (21) and (22), we have

$$u(s) = -\frac{1}{s}F_y y(s) - F_C(s)e^{A\tau}(sI - A + KC)^{-1}Ky(s) + F_C(s)[(I - e^{-(sI-A)^{\tau}})(sI-A)^{-1}B + e^{-(sI-A)\tau}(sI - A + KC)^{-1}]u(s) \quad (23)$$

It follows from (23) that the controller transfer function matrix is expressed as (16). □

Remark 1: Note that (16) is not a factorization by proper and stable matrices, as the matrix (17) is not stable. Stable factorization is not required for our purpose.

The following result can easily be obtained from the above result.

Proposition 2: Consider the output feedback integral controller shown in Fig. 3. The sensitivity matrix at the plant input side

$$\Sigma_\tau(s) \triangleq [I - C_\tau(s)C(sI-A)^{-1}B]^{-1} \quad (24)$$

can be expressed as

$$\Sigma_\tau(s) = \Sigma(s)\overline{V}_\tau(s) \quad (25)$$

where

$$\Sigma(s) \triangleq \left[I + \left(\frac{1}{s}F_y C + F_x\right)(sI-A)^{-1}B\right]^{-1} \quad (26)$$

and $\bar{V}_\tau(s)$ is defined in (18).

It can be shown that the matrix (26) coincides with the sensitivity matrix for the state feedback without the time-delay ($\tau = 0$). For the state feedback with the time-delay $\tau > 0$, we can easily show the following result.

Proposition 3: Consider the state feedback integral controller for the time-delay plant shown in Fig. 2. The sensitivity matrix at the plant side can be expressed as

$$\Sigma_\tau^{SF}(s) = \Sigma(s)\bar{V}_\tau^{SF}(s) \quad (27)$$

where

$$\bar{V}_\tau^{SF}(s) \triangleq I + F_C(s)(I - e^{-(sI-A)\tau})(sI-A)^{-1}B \quad (28)$$

and $\Sigma(s)$ is defined in (26).

For efficient determination of the parameters of the output feedback integral controller in Fig. 3, we show that the standard LTR procedure for LQG controllers can be used for the LTR design, targeting the state feedback integral controller shown in Fig. 2.

Assume that the observer gain matrix, K, in (10) is determined as a Kalman filter gain matrix [12]. Introduce the Riccati equation

$$P_f A' + AP_f - P_f C'\Theta^{-1}CP_f + \rho BB' = 0 \quad (29)$$

where Θ is a positive definite matrix and ρ is a positive scalar parameter. The Kalman gain matrix is given by

$$K = P_f C'\Theta^{-1} \quad (30)$$

For the above choice of the observer gain matrix, we obtain the following result.

Proposition 4: Consider the output feedback integral controller shown in Fig. 3. Assume that the observer with the gain matrix K, determined by (29) and (30), is used in the controller. Then the sensitivity matrix (25) at the plant input side satisfies the asymptotic relation

$$\lim_{\rho\to\infty} \Sigma_\tau(s) = \Sigma_\tau^{SF}(s) \quad (31)$$

where $\Sigma_\tau^{SF}(s)$ is the sensitivity matrix defined in (27) for the state feedback integral controller with the simplified predictor.

Proof: Let $K(\rho)$ denote the Kalman filter gain matrix obtained from (29) and (30). It is well-known [12] that, for sufficiently large ρ, the Kalman filter gain matrix can be expressed as

$$K(\rho) \simeq \rho^{1/2}B\Theta^{-1/2} \quad (32)$$

Noting that

$$[sI - A + K(\rho)C]^{-1}B = (sI-A)^{-1}B \times [I + \rho^{1/2}\Theta^{-1/2}C(sI-A)^{-1}B]^{-1} \to 0\,(\rho\to\infty) \quad (33)$$

we can easily show that

$$\lim_{\rho\to\infty} \overline{V}_\tau(s) = \overline{V}_\tau^{SF}(s) \quad (34)$$

The relation (31) follows from (25), (27) and (34). □

The above result suggests that the state feedback integral controller with the simplified predictor given in *Proposition 3* can be regarded as a recoverable target for the formal LTR procedure at the plant input side. From this result, we can use the LTR procedure to determine the controller parameters considering the feedback properties at the plant input side. First, determine the target by choosing the state feedback matrices, F_x and F_y, such that the state feedback controller with the simplified predictor has desirable feedback properties. The expression (27) can be used in this step. Then the scalar parameter, ρ, is determined such that the output feedback controller with F_x, F_y and $K(\rho)$ has feedback properties sufficiently close to the target properties.

Remark 2: Unlike the targets of the standard LQG/LTR design, for example, LQ regulators, The target does not possess guaranteed stability margins (27) can conveniently be used to find an appropriate target. Note that the state feedback integral controller without the time-delay possesses the large stability margins if the feedback gain matrix, F, defined in (7) for the extended system (2), is determined as an optimal feedback gain matrix for the quadratic performance index (4). Then the sensitivity matrix $\Sigma(s)$, defined in (26), has a nice property. (27), where $\bar{V}_\tau^{SF}(s)$, defined in (28), explicitly represents the effect of the time-delay, is useful for determining the target feedback properties.

Remark 3: The controller using the optimal state predictor for the extended system can be designed by the LTR technique [9]. Then the target for the LTR at the plant input side is a state feedback integral controller using the state predictor. It can be shown that the target control system has the sensitivity matrix given by

$$\Sigma_\tau^{OSF}(s) = \Sigma(s)\overline{V}_\tau^{OSF}(s) \tag{35}$$

where

$$\begin{aligned}\bar{V}_\tau^{OSF}(s) \triangleq I + \left(F_x + \frac{1}{s}F_yC\right)(I - e^{-(sI-A)\tau}) \\ \times (sI-A)^{-1}B - \frac{1}{s}e^{-\tau s}F_yC\left(\int_0^\tau e^{A\sigma}d\sigma\right)B\end{aligned} \tag{36}$$

3.2 Plant Output Side

For the plant input side, the left factorization form (16) of the controller transfer function matrix simplifies the discussion. To discuss the sensitivity property at the plant output side (the point P_y in Fig. 3), it is convenient to introduce a right factorization form of the controller transfer function matrix. The right factorization is given as follows.

Proposition 5: The transfer function matrix, $C_\tau(s)$, from the measured output $y(t)$ to the input $u(t)$ in the output feedback integral controller shown in Fig. 3, can be expressed as

$$C_\tau(s) = -U_\tau(s)V_\tau^{-1}(s) \tag{37}$$

where

$$U_\tau(s) \triangleq F(s)[sI - A + BF(s)]^{-1}K_B(s) + \frac{1}{s}F_yV_\tau^*(s) \tag{38}$$

$$V_\tau(s) \triangleq V_\tau^*(s) + e^{-\tau s}C[sI - A + BF(s)]^{-1}K_B(s) \tag{39}$$

$$F(s) \triangleq \frac{1}{s}F_yC + F_x \tag{40}$$

$$V_\tau^*(s) \triangleq I + C(I - e^{-(sI-A)\tau})(sI-A)^{-1}K \tag{41}$$

$$K_B(s) \triangleq e^{A\tau}K - \frac{1}{s}BF_yV_\tau^*(s) \tag{42}$$

Proof: The derivation requires long matrix calculations. See Appendix. □

Using the above factorization, we have the following result.

Proposition 6: Consider the output feedback integral controller shown in Fig. 3. The sensitivity matrix at the plant output side

$$\Pi_\tau(s) \triangleq [I - C(sI-A)^{-1}BC_\tau(s)]^{-1} \tag{43}$$

can be expressed as

$$\Pi_\tau(s) = V_\tau(s)\Pi(s) \tag{44}$$

where

$$\Pi(s) \triangleq [I + C(sI-A)^{-1}K]^{-1} \tag{45}$$

is the sensitivity matrix for the observer and $V_\tau^*(s)$ is defined in (41).

Proof: Using the right factorization of the controller transfer function matrix, we can write the sensitivity matrix as

$$\Pi(s) = V_\tau(s)[I + C(sI-A)^{-1}K + \Xi_\tau(s)]^{-1} \tag{46}$$

where

$$\begin{aligned}\Xi_\tau(s) \triangleq & -C(sI-A)^{-1}e^{-(sI-A)\tau}K \\ & + e^{-\tau s}C[sI - A + BF(s)]^{-1}K_B(s) \\ & + e^{-\tau s}C(sI-A)^{-1}BF(s)C[sI - A \\ & + BF(s)]^{-1}K_B(s) \\ & + \frac{e^{-\tau s}}{s}C(sI-A)^{-1}BF_yV_\tau^*(s)\end{aligned} \tag{47}$$

Simple matrix calculation shows that

$$\Xi_\tau(s) = e^{-\tau s}C(sI-A)^{-1}\left[-e^{A\tau}K + K_B(s) + \frac{1}{s}F_yV_\tau^*(s)\right] \tag{48}$$

Using (42) in (48), we have

$$\Xi_\tau(s) = 0 \tag{49}$$

The expression (44) follows from (46) and (49). □

The standard LQG/LTR method for the plant output side utilizes a scalar parameter related to the weighting matrices of a quadratic performance index as a recovery parameter. For the output feedback controller shown in Fig. 3, we clarify the asymptotic sensitivity property at the plant output side using Preposition 6.

Let $F(q)$ denote the feedback gain matrix (6) determined by the performance index (4) with the weighting matrix

$$Q = qI \tag{50}$$

where q is a positive scalar parameter.

For the weighting matrix (50), the asymptotic sensitivity matrix with respect to the parameter q is given by the following result.

Proposition 7: Consider the output feedback integral controller shown in Fig. 3. The sensitivity matrix at the plant input output side given by (44) satisfies the asymptotic relation

$$\Pi_\tau^*(s) \triangleq \lim_{q\to\infty} \Pi_\tau(s) = (1 - e^{-\tau s})V_\tau^*(s)\Pi(s) \tag{51}$$

where $V_\tau^*(s)$ and $\Pi(s)$ are defined in (41) and (45), respectively.

Proof: Using the well known result for the asymptotic property of the optimal feedback gain matrix [12], we have

$$\lim_{q\to\infty} q^{-1/2}R^{1/2}F(q) = H \tag{52}$$

where H is the matrix defined in (3). Define the partitions of $F(q)$ as

$$F(q) = [F_x(q) \quad F_y(q)] \tag{53}$$

It follows from (3) and (52) that

$$\lim_{q\to\infty} q^{-1/2}R^{1/2}F_x(q) = 0, \quad \lim_{q\to\infty} q^{-1/2}R^{1/2}F_y(q) = I \tag{54}$$

For sufficiently large values of $q, F_x(q)$ and $F_y(q)$ can be approximated by

$$F_x(q) \simeq 0, \quad F_y(q) \simeq q^{1/2}R^{1/2} \tag{55}$$

Using the above asymptotic properties, we derive the asymptotic expression of the matrix $V_\tau(s)$.

$$V_\tau^A(s) \triangleq \lim_{q\to\infty} C[sI - A + BF(s)]^{-1}e^{A\tau}K \tag{56}$$

$$V_\tau^B(s) \triangleq \lim_{q\to\infty} C[sI - A + BF(s)]^{-1}\frac{q^{1/2}}{s}BR^{-1/2}V_\tau^*(s) \tag{57}$$

It then follows from (39) that

$$\lim_{q\to\infty} V_\tau(s) = V_\tau^*(s) + V_\tau^A(s) - e^{-\tau s}V_\tau^B(s) \tag{58}$$

Using (55) in (56), we obtain

$$\begin{aligned} V_\tau^A(s) = \lim_{q\to\infty} &\left[I + \frac{q^{1/2}}{s}C(sI - A)^{-1}BR^{-1/2}\right]^{-1} \\ &\times C(sI - A)^{-1}e^{A\tau}K = 0 \end{aligned} \tag{59}$$

From (55) and (57), we have

$$\begin{aligned} V_\tau^B(s) &= \lim_{q\to\infty} \frac{q^{1/2}}{s}C(sI - A)^{-1}BR^{-1/2} \\ &\quad \times \left[I + \frac{q^{1/2}}{s}C(sI - A)^{-1}BR^{-1/2}\right]^{-1} V_\tau^*(s) \\ &= V_\tau^*(s) \end{aligned} \tag{60}$$

Note that the non-singularity assumption on $C(sI - A)^{-1}B$ is used in (59) and (60). The expression (51) follows from (44), (58), (59) and (60). □

Unlike the plant input side, it seems difficult to give clear system-theoretic meaning on the recoverable sensitivity matrix (51). It is well-known that, for the standard LQG/LTR method, the feedback properties achieved at the plant output side are dual to those at the plant input side. For the optimal Davison type integral controller, the duality is not apparent but has been found as remarked below.

Remark 4: Wu *et al.* [9] have shown that, for the Davison type integral controller using the optimal predictor, the asymptotic sensitivity matrix achieved at the plant output side is given by

$$\Pi_\tau^{OP}(s) = V_\tau^{OP}(s)\Pi(s) \tag{61}$$

where

$$\begin{aligned} V_\tau^{OP}(s) \triangleq I + C(I - e^{-(sI-A)\tau})(sI - A)^{-1}K - e^{-\tau s} \\ \times \left[I + C\left(\int_0^\tau e^{A\sigma}d\sigma\right)K\right] \end{aligned} \tag{62}$$

In addition, it has been found that the above sensitivity matrix is related to an estimation problem, which suggests that hidden duality exists for the controller using the optimal state predictor.

We conjecture that a hidden duality exists for the controller with the simplified predictor. For the moment, however, we cannot find a dual estimation problem related to the sensitivity matrix (51). Although the system-theoretic meaning of the recoverable target is not clear, *Proposition 7* gives the LTR procedure focused on the plant output side. First, determine the observer gain matrix, K, such that the sensitivity matrix, $\Pi_\tau^*(s)$, given by (51), and the complementary sensitivity matrix $I - \Pi_\tau^*(s)$, have desirable feedback properties. Then the scalar parameter, q, is determined such that the output feedback controller with $F_x(q)$, $F_y(q)$ and K has the feedback properties sufficiently close to feedback properties given by $\Pi_\tau^*(s)$ and $I - \Pi_\tau^*(s)$.

Remark 5: If the observer gain matrix K is determined as a Kalman filter gain matrix, the sensitivity matrix, $\Pi(s)$, defined in (45) has a nice property. The term $(1-e^{-\tau s})V_\tau^*(s)$ in (51) represents the effect of the time-delay, τ, on the sensitivity matrix.

Remark 6: The expression (51) implies that $\Pi_\tau(0)=0$ for $\tau \geq 0$, which reflects the integral action.

Remark 7: In the delay-free case ($\tau=0$), (51) reduces to $\Pi_0(s)\equiv 0$, that is, zero sensitivity is achieved at the plant output side as $q \to \infty$. This is a unique characteristic of the Davison type controller. The standard LQG/LTR design cannot provide this characteristic.

4. Illustrative Example

The purpose of this section is to provide numerical illustration of the effect of simplifying the predictor on the feedback properties. Consider a simple heat exchanger [13] given by

$$G(s) = \frac{e^{-5s}}{(1+10s)(1+60s)} \tag{63}$$

For the above plant, the feedback properties of the integral controller using the simplified predictor are compared with those of the optimal design [9]. As the Davison type integral controller has distinctive feedback properties at the plant output side as pointed in *Remark 7*, we give the results only for the plant output side.

For the realization

$$A = \begin{bmatrix} -0.017 & -0.017 \\ 0 & -0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad C = [1 \quad 0] \tag{64}$$

the Kalman filter is designed for $\Theta=1$ and $\rho=100$ in (29). The same Kalman filter is used for both integral controller designs. The comparison of the asymptotic feedback properties of the two designs as $q\to\infty$ is shown in Fig. 4. For the optimal design, the gain characteristics are calculated using (61) and (62). The two designs provide almost the same feedback properties in the low frequency region. In the frequency region higher than 0.1 rad/sec, however, significant difference exists between the complementary sensitivity functions. It is tempting to conclude that the controller using the simplified predictor has a better robustness property against uncertainty in a high frequency region. Special care should be taken in interpreting the difference in Fig. 4. It should be noted that Fig. 4 shows the limiting characteristics that converge pointwise in s. For finite q, the gain characteristics of the complementary sensitivity functions for both designs decrease with 40 dB/dec in a frequency region higher than a frequency determined by q as shown in Fig. 5 for $q=10^5$. It is seen that the two designs provide almost the same feedback properties for all frequencies. It can be concluded that, as far as the designs with finite q are concerned, the use of the simplified predictor does not provide better robustness, but provides almost the same robustness property as the optimal controller.

5. Conclusion

A new integral controller for time-delay plants has been proposed. The controller utilizes a simplified predictor, which reduces real-time computation required for the optimal design. As a systematic design procedure, the application of the LTR technique is proposed. For a simple plant, the achievable feedback properties are numerically compared with those of the optimal controller. The results suggest that the new controller can achieve feedback properties nearly the same as in the optimal design.

The method used in [8], based on the partial recovery technique [11], can be used to extend the present result to

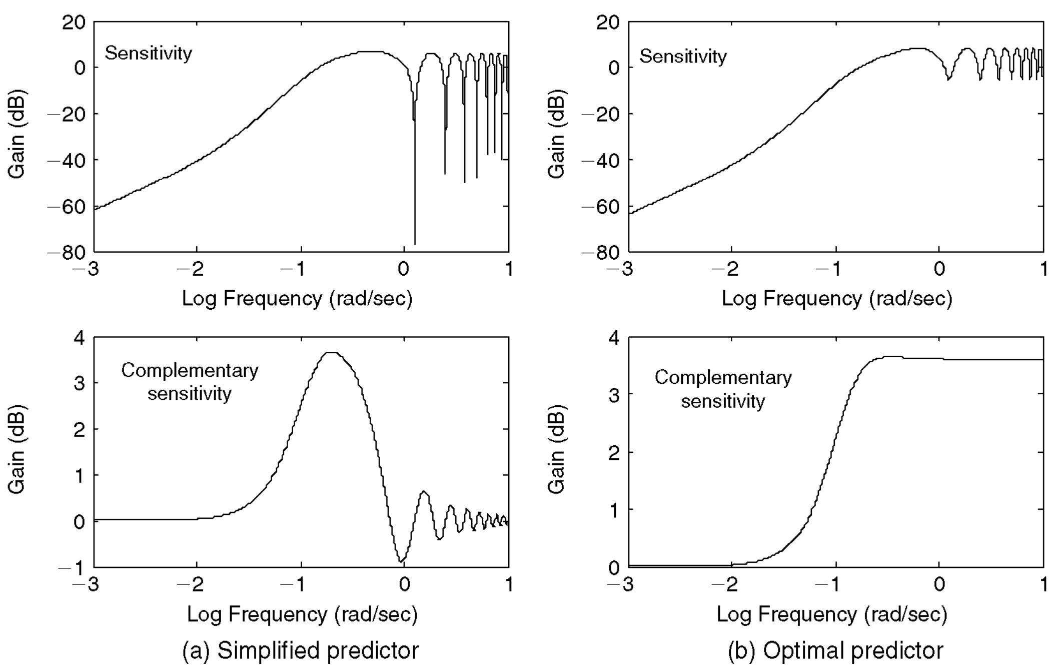

Figure 4. Comparison of asymptotic feedback properties.

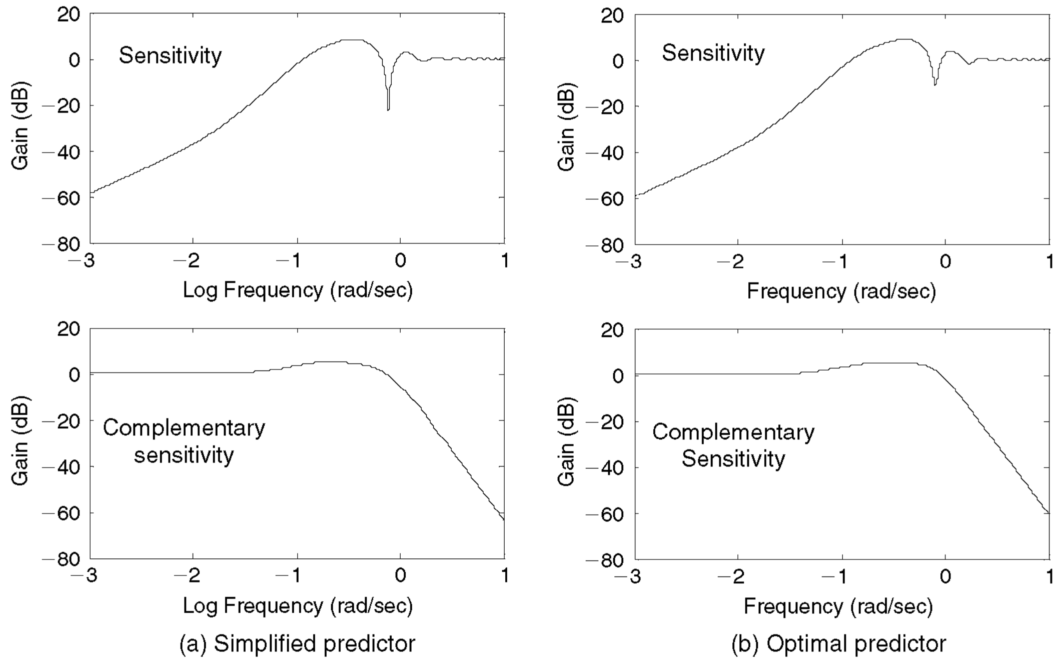

Figure 5. Comparison of feedback properties for $q = 10^5$.

the case in which the lumped part of the plant has unstable zeros.

References

[1] E.J. Davison & I.J. Ferguson, The design of controllers for the multivariable robust servo-mechanism problem using parameter optimization methods, *IEEE Trans. on Automatic Control*, *26*(1), 1981, 93–110.

[2] K. Watanabe, Predictor control having the Y-type structure for systems with delay in input, *Trans. of Society of Instrument and Control Engineers*, *21*(9), 1985, 928–933.

[3] O.J.M. Smith, Closer control of loops with deadtime, *Chemical Engineering Progress*, *53*, 1957, 217–219.

[4] A. Saberi, P. Sannuti, & B.M. Chen, *Loop transfer recovery* (New York: Springer-Verlag, 1993).

[5] G. Stein & M. Athans, The LQG/LTR procedure for multi-variable feedback control design, *IEEE Trans. on Automatic Control*, *32*(2), 1987, 105–114.

[6] W.H. Kwon & S.J. Lee, LQG/LTR methods for linear systems with delay in state, *IEEE Trans. on Automatic Control*, *33*(7), 1988, 681–687.

[7] S. J. Lee, W. H. Kwon, & S.W. Kim, LQG/LTR methods for linear input-delayed systems, *International Journal of Control*, *47*(5), 1988, 1179–1194.

[8] J. Wu, T. Ishihara, & H. Inooka, Loop transfer recovery technique for time-delay systems with non-minimum phase lumped part, *Trans.* of *Institute of Systems*, *Control and Information Engineers*, *9*(5), 1996, 236–245.

[9] J. Wu, T. Ishihara, & X.G. Wang, Design of predictor-based integral controllers via LTR for plant output side, *Proc. of the 4th Asian Control Conf.*, Singapore, 2002, 477–482.

[10] Z. Zhang & J.S. Freudenberg, Loop transfer recovery for non-minimum phase plants, *IEEE Trans. on Automatic Control*, *33*(7), 1990, 547–553.

[11] J.B. Moore & L. Xia, Loop recovery and robust state feedback design, *IEEE Trans. on Automatic Control*, *32*(6), 1987, 512–517.

[12] B.D.O. Anderson & J. B. Moore, *Optimal control* (Englewood Cliffs, NJ: Prentice-Hall, 1990).

[13] G.F. Franklin, J.D. Powell, & A. Emami-Naeini, *Feedback control of dynamic systems* (Reading, MA: Addison-Wiseley, 1986).

Appendix: Proof of *Proposition 6*

Define the matrices

$$\tilde{A} \triangleq \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{B} \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \tilde{C} \triangleq [C \quad 0], \quad \tilde{K} \triangleq \begin{bmatrix} K \\ 0 \end{bmatrix}, \tag{A.1}$$

$$\tilde{F}(s) \triangleq [F_C(s) \quad F_y], \quad \tilde{I} \triangleq \begin{bmatrix} 0 \\ I \end{bmatrix}$$

where the matrix $F_C(s)$ is defined in (14). Then the controller transfer function (16) can be expressed as

$$\begin{aligned} C_\tau(s) =& -[I + \tilde{F}(s)(I - e^{-(sI-A)\tau})(sI - \tilde{A})^{-1}\tilde{B} \\ & + \tilde{F}(s)e^{-(sI-\tilde{A})\tau}(sI - \tilde{A} + \tilde{K}\tilde{C})^{-1}\tilde{B}]^{-1} \\ & \times \tilde{F}(s)e^{\tilde{A}\tau}(sI - \tilde{A} + \tilde{K}\tilde{C})^{-1}(\tilde{K} + \tilde{I}) \end{aligned} \tag{A.2}$$

which can be rewritten as

$$\begin{aligned} C_\tau(s) =& -\tilde{F}(s)\{I + [I + (I - e^{-(sI-\tilde{A})\tau})(sI - \tilde{A})^{-1}\tilde{K}\tilde{C}] \\ & \times (sI - \tilde{A} + \tilde{K}\tilde{C})^{-1}\tilde{B}\tilde{F}(s)\}^{-1} \\ & \times e^{\tilde{A}\tau}(sI - \tilde{A} + \tilde{K}\tilde{C})^{-1}(\tilde{K} + \tilde{I}) \end{aligned} \tag{A.3}$$

Defining the matrix

$$\tilde{D}(s) \triangleq I + (I - e^{-(sI-\tilde{A})\tau})(sI - \tilde{A})^{-1}\tilde{K}\tilde{C} \tag{A.4}$$

we can write (A.3) as

$$\begin{aligned} C_\tau(s) =& -\tilde{F}(s)[\tilde{D}^{-1}(s) + (sI - \tilde{A} \\ & + \tilde{K}\tilde{C})^{-1}\tilde{B}\tilde{F}(s)]^{-1}\tilde{D}^{-1}(s) \\ & \times e^{\tilde{A}\tau}(sI - \tilde{A} + \tilde{K}\tilde{C})^{-1}(\tilde{K} + \tilde{I}) \end{aligned} \tag{A.5}$$

Multiplying $(sI-\tilde{A})$ from the left and $\tilde{D}^{-1}(s)$ from the right of (A.4) yields

$$(sI-\tilde{A}+\tilde{K}\tilde{C})\tilde{D}^{-1}(s)=(sI-\tilde{A})+e^{-(sI-\tilde{A})\tau}\tilde{K}\tilde{C}\tilde{D}^{-1}(s) \tag{A.6}$$

It follows from (A.5) and (A.6) that

$$\begin{aligned}C_\tau(s)=&-\tilde{F}(s)\{[sI-\tilde{A}+\tilde{B}\tilde{F}(s)]\\&+e^{-(sI-\tilde{A})\tau}\tilde{K}\tilde{C}\tilde{D}^{-1}(s)\}^{-1}\\&\times(sI-\tilde{A}+\tilde{K}\tilde{C})\tilde{D}^{-1}(s)e^{\tilde{A}\tau}(sI-\tilde{A}\\&+\tilde{K}\tilde{C})^{-1}(\tilde{K}+\tilde{I})\end{aligned} \tag{A.7}$$

Introduce the matrix

$$\tilde{E}(s)\triangleq I+\tilde{K}\tilde{C}(I-e^{-(sI-\tilde{A})\tau})(sI-\tilde{A})^{-1} \tag{A.8}$$

which satisfies the relation:

$$\tilde{E}^{-1}(s)(sI-\tilde{A}+\tilde{K}\tilde{C})=(sI-\tilde{A})+\tilde{E}^{-1}(s)\tilde{K}\tilde{C}e^{-(sI-\tilde{A})\tau} \tag{A.9}$$

as in the identity (A.6) for (A.4). It follows from (A.6) and (A.9) that:

$$\begin{aligned}&e^{(sI-\tilde{A})\tau}(sI-\tilde{A}+\tilde{K}\tilde{C})\tilde{D}^{-1}(s)\\&\quad=e^{(sI-\tilde{A})\tau}(sI-\tilde{A})+\tilde{K}\tilde{C}\tilde{D}^{-1}(s)\end{aligned} \tag{A.10}$$

$$\begin{aligned}&\tilde{E}^{-1}(s)(sI-\tilde{A}+\tilde{K}\tilde{C})e^{(sI-\tilde{A})\tau}\\&\quad=(sI-\tilde{A})e^{(sI-\tilde{A})\tau}+\tilde{E}^{-1}(s)\tilde{K}\tilde{C}\end{aligned} \tag{A.11}$$

Note that the matrix identities

$$\begin{aligned}&\tilde{K}\tilde{C}\tilde{D}^{-1}(s)=\tilde{E}^{-1}(s)\tilde{K}\tilde{C},\\&e^{(sI-\tilde{A})\tau}(sI-\tilde{A})=(sI-\tilde{A})e^{(sI-\tilde{A})\tau}\end{aligned} \tag{A.12}$$

hold. It follows from (A.10), (A.11) and (A.12) that:

$$\begin{aligned}&e^{(sI-\tilde{A})\tau}(sI-\tilde{A}+\tilde{K}\tilde{C})\tilde{D}^{-1}(s)\\&\quad=\tilde{E}^{-1}(s)(sI-\tilde{A}+\tilde{K}\tilde{C})e^{(sI-\tilde{A})\tau}\end{aligned} \tag{A.13}$$

which can be rewritten as

$$\begin{aligned}&\tilde{D}^{-1}(s)e^{\tilde{A}\tau}(sI-\tilde{A}+\tilde{K}\tilde{C})^{-1}\\&\quad=(sI-\tilde{A}+\tilde{K}\tilde{C})^{-1}e^{\tilde{A}\tau}\tilde{E}^{-1}(s)\end{aligned} \tag{A.14}$$

Substituting (A.14) into (A.7) yields

$$\begin{aligned}C_\tau(s)=&-\tilde{F}(s)[sI-\tilde{A}+\tilde{B}\tilde{F}(s)\\&+e^{-(sI-\tilde{A})\tau}\tilde{K}\tilde{C}\tilde{D}^{-1}(s)]^{-1}e^{\tilde{A}\tau}\tilde{E}^{-1}(s)(\tilde{K}+\tilde{I})\end{aligned} \tag{A.15}$$

Define the matrix

$$\tilde{\Delta}(s)\triangleq I+\tilde{C}(I-e^{-(sI-\tilde{A})\tau})(sI-\tilde{A})^{-1}\tilde{K} \tag{A.16}$$

Then the matrix identities

$$\tilde{C}\tilde{D}^{-1}(s)=\tilde{\Delta}^{-1}(s)\tilde{C}\quad \tilde{E}^{-1}(s)\tilde{K}=\tilde{K}\tilde{\Delta}^{-1}(s) \tag{A.17}$$

hold. In addition, it follows from (A.1) and (A.8) that

$$\tilde{E}^{-1}(s)\tilde{I}=\tilde{I} \tag{A.18}$$

Substituting (A.17) and (A.18) into (A.15) yields

$$\begin{aligned}C_\tau(s)=&-\tilde{F}(s)[sI-\tilde{A}+\tilde{B}\tilde{F}(s)\\&+e^{-(sI-\tilde{A})\tau}\tilde{K}\tilde{\Delta}^{-1}(s)\tilde{C}]^{-1}e^{\tilde{A}\tau}(\tilde{K}\tilde{\Delta}^{-1}(s)+\tilde{I})\end{aligned} \tag{A.19}$$

Noting that

$$e^{\tilde{A}\tau}\tilde{I}=\tilde{I} \tag{A.20}$$

holds by definition, we can rewrite (A.19) as

$$\begin{aligned}C_\tau(s)=&-\tilde{F}(s)\{sI-\tilde{A}-e^{-(sI-\tilde{A})\tau}\tilde{I}\tilde{C}\\&+\tilde{B}\tilde{F}(s)+e^{-(sI-\tilde{A})\tau}\\&\times[\tilde{K}\tilde{\Delta}^{-1}(s)+\tilde{I}]\tilde{C}\}^{-1}e^{\tilde{A}\tau}[\tilde{K}\tilde{\Delta}^{-1}(s)+\tilde{I}]\\=&-\tilde{F}(s)[sI-\tilde{A}-e^{-s\tau}\tilde{I}\tilde{C}+\tilde{B}\tilde{F}(s)]^{-1}e^{\tilde{A}\tau}\\&\times[\tilde{K}\tilde{\Delta}^{-1}(s)+\tilde{I}]\{\tilde{\Delta}(s)+\tilde{C}[sI-\tilde{A}-e^{-s\tau}\tilde{I}\tilde{C}\\&+\tilde{B}\tilde{F}(s)]^{-1}e^{-(sI-\tilde{A})\tau}[\tilde{K}\tilde{\Delta}^{-1}(s)+\tilde{I}]\}^{-1}\end{aligned} \tag{A.21}$$

Define

$$\begin{aligned}U_\tau(s)\triangleq&-\tilde{F}(s)[sI-\tilde{A}-e^{-s\tau}\tilde{I}\tilde{C}\\&+\tilde{B}\tilde{F}(s)]^{-1}e^{\tilde{A}\tau}[\tilde{K}\tilde{\Delta}^{-1}(s)+\tilde{I}]\end{aligned} \tag{A.22}$$

$$\begin{aligned}V_\tau(s)\triangleq&\tilde{\Delta}(s)+\tilde{C}[sI-\tilde{A}-e^{-s\tau}\tilde{I}\tilde{C}\\&+\tilde{B}\tilde{F}(s)]^{-1}e^{-(sI-\tilde{A})\tau}[\tilde{K}\tilde{\Delta}^{-1}(s)+\tilde{I}]\end{aligned} \tag{A.23}$$

Then the controller transfer function matrix (A.19) is given in the right factorization form as

$$C_\tau(s)=U_\tau(s)V_\tau^{-1}(s) \tag{A.24}$$

In the following, we simplify the expressions of (A.22) and (A.23). Noting that

$$\tilde{\Delta}(s)=V_\tau^*(s) \tag{A.25}$$

holds from (41), (A.1) and (A.16), we have

$$\tilde{K}+\tilde{I}\tilde{\Delta}(s)=\begin{bmatrix}K\\V_\tau^*(s)\end{bmatrix} \tag{A.26}$$

In addition, we find that

$$
\begin{aligned}
&[sI - \tilde{A} - e^{-\tau s}\tilde{I}\tilde{C} + \tilde{B}\tilde{F}(s)]^{-1} \\
&\quad = \begin{bmatrix} sI - A + BF_C(s) & BF_y \\ -e^{-\tau s}C & sI \end{bmatrix}^{-1} \\
&\quad = \begin{bmatrix} I & 0 \\ e^{-\tau s}\frac{1}{s}C & I \end{bmatrix} \begin{bmatrix} [sI - A + BF(s)]^{-1} & 0 \\ 0 & \frac{1}{s}Is \end{bmatrix} \\
&\qquad \times \begin{bmatrix} I & -\frac{1}{s}BF_y \\ 0 & I \end{bmatrix} \qquad \text{(A.27)}
\end{aligned}
$$

where $F(s)$ is defined in (40). From (A.26) and (A.27), we have:

$$
\begin{bmatrix} I & -\frac{1}{s}BF_y \\ 0 & I \end{bmatrix} e^{\tilde{A}\tau}[\tilde{K} + \tilde{I}\tilde{\Delta}(s)] = \begin{bmatrix} K_B(s) \\ V_\tau^*(s) \end{bmatrix} \qquad \text{(A.28)}
$$

where $K_B(s)$ is defined in (42). Using (A.25), (A.27) and (A.28) in (A.22) and (A.23), we can rewrite (38) and (39) as

$$
\begin{aligned}
U_\tau(s) &= -[F_C(s) \quad F_y] \begin{bmatrix} I & 0 \\ e^{-\tau s}\frac{1}{s}C & I \end{bmatrix} \\
&\quad \times \begin{bmatrix} [sI - A + BF(s)]^{-1} & 0 \\ 0 & \frac{1}{s}I \end{bmatrix} \begin{bmatrix} K_B(s) \\ V_\tau^*(s) \end{bmatrix} \\
&= F(s)[sI - A + BF(s)]^{-1}K_B(s) + \frac{1}{s}F_yV_\tau^*(s), \qquad \text{(A.29)}
\end{aligned}
$$

$$
\begin{aligned}
V_\tau(s) = &V_\tau^*(s) + e^{-\tau s}[C \quad 0] \begin{bmatrix} I & 0 \\ e^{-\tau s}\frac{1}{s}C & I \end{bmatrix} \\
&\times \begin{bmatrix} [sI - A + BF(s)]^{-1} & 0 \\ 0 & \frac{1}{s}I \end{bmatrix} \begin{bmatrix} K_B(s) \\ V_\tau^*(s) \end{bmatrix} \\
&= V_\tau^*(s) + e^{-\tau s}C[sI - A + BF(s)]^{-1}K_B(s) \qquad \text{(A.30)}
\end{aligned}
$$

which proves the proposition.

Biographies

Jingwei Wu received his M.S. degree from The Harbin Institute of Technology, Harbin, China, in 1980, and his Ph.D. from The Institute of Technology, Tohoku University, Sendai, in 1996. He joined The Harbin University of Science and Technology in 1981, where he served as a lecturer until 1991. He was with Minolta Japan and the National Research Council of Canada from 1996 to 2003 as a senior engineer and technical officer. His research interests are multivariable robust control, system identification, and process control. Since 2004 he has been a general manager in Dehua International Mines Group Inc. in Canada.

Tadashi Ishihara received his B.E., M.E., and Ph.D. degrees, all in electrical engineering, from Tohoku University, Sendai, Japan, in 1972, 1974 and 1979, respectively. From 1977 to 1987 he was an Assistant Professor in The Department of Electrical Engineering, Tohoku University. From 1987 to 1993 he was an Associate Professor in The Department of Mechanical Engineering, Tohoku University. From 1993 to 2003, he was an Associate Professor in Graduate School of Information Sciences, Tohoku University. Currently, he is a Professor in the Faculty of Science and Technology, Fukushima University, Fukushima, Japan. His current research interests are robust control, stochastic adaptive control, and computer aided control systems design based on the principle of matching.

Q.M. Jonathan Wu received his Ph.D. in electrical engineering from The University of Wales, Swansea, U.K., in 1990. In 1995 he joined The National Research Council of Canada, where he became a senior research officer and group leader. He is currently a Professor in The Department of Electrical and Computer Engineering at The University of Windsor, Canada. Dr. Wu was recently named as the Canada Research Chair (CRC) in Automotive Sensors and Sensing Systems. He has published more than 100 scientific papers in the areas of computer vision, neural networks, fuzzy systems, robotics, micro-sensors and actuators, and integrated micro-systems. His current research interests include 3D image analysis, active video object tracking and extraction, vision-guided robotics, sensor analysis and fusion, wireless sensor network, and integrated micro-systems. Dr. Wu is an associate editor for IEEE Transaction on Systems, Man, and Cybernetics (part A). Dr. Wu is also on the editorial board of *The Journal of Control and Intelligent Systems*.