PATTERN RECOGNITION BY AFFINE LEGENDRE MOMENT INVARIANTS

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ABSTRACT

Affine moment invariants are important shape descriptors in pattern recognition and computer vision. Existing affine invariants methods are based on geometric and complex moments. In this paper, we propose a set of affine invariants extracted from Legendre moments. These invariants are derived by the relationship between the Legendre moment of the affine transformed image and that of the original image. The performance of the proposed descriptor is evaluated with a set of binary and gray images. Experimental results show that the proposed method behaves better than existing methods in terms of pattern recognition accuracy.

***Index Terms*—**Affine invariants, affine transformation, Legendre moments, pattern recognition.

1. INTRODUCTION

Affine moment invariants (AMIs) are useful features since they are invariant to general linear transformations of an image [1], [11]. The construction of affine moment invariants has been extensively investigated in the past decades. Existing methods can be generally classified into two categories: (1) direct method, and (2) image normalization. Amongst the direct methods, Reiss [2] and Flusser et al. [3] derived AMIs based on the theory of algebraic invariants and tensor techniques. Normalization is an alternative approach to derive moment invariants. Rothe et al. [4] first introduced the concept of affine normalization. In their work, two different affine decompositions were used. The first one, known as the XSR decomposition, consists of one skew, anisotropic scaling and rotation. The second one is the XYS and is based on two skews and anisotropic scaling. These affine decompositions have been studied by Zhang et al. [5] who pointed out that both decompositions lead to some ambiguities. Pei and Lin [6] presented an affine normalization for asymmetrical objects. Hosny [7] proposed an exact method for the computation of affine moment invariants.

All existing methods derive affine invariants based on geometric and complex moments. Since the kernel functions of both geometric moments and complex moments are not orthogonal, the image representation contains some information redundancy and noise sensitivity. This motivates us to use the orthogonal moments for the construction of affine invariants.

2. LEGENDRE MOMENTS

The two-dimensional (2-D) Legendre moment of order (p+q) of an image intensity function $f(x, y)$ is defined as [1]

$$L_{pq}^{(f)} = \int_{-1}^{1}\int_{-1}^{1} P_p(x)P_q(y)f(x,y)dxdy, , \tag{1}$$

where $P_p(x)$ is the pth order orthogonal Legendre polynomial defined by:

$$P_p(x) = \sum_{k=0}^{p} c_{p,k}x^k, \tag{2}$$

where

$$c_{p,k} = \begin{cases} \sqrt{\frac{2p+1}{2}} \frac{(-1)^{\frac{p-k}{2}}(p+k)!}{2^p\left(\frac{p-k}{2}\right)!\left(\frac{p+k}{2}\right)!k!}, & p-k=\text{even} \\ 0, & p-k=\text{odd} \end{cases}. \tag{3}$$

It can be deduced from (2) that

$$x^p = \sum_{k=0}^{p} d_{p,k}P_k(x), \tag{4}$$

where $D_M = (d_{p,k})$, $0 \le k \le p \le M$, is the inverse of the lower triangular matrix $C_M = (c_{p,k})$. The elements of D_M are given by [8]

$$d_{p,k} = \begin{cases} \frac{\sqrt{\frac{2}{2k+1}}2^{\frac{3k-p}{2}}}{\left(\frac{p-k}{2}\right)!\prod_{j=1}^{(p-k)/2}(2k+2j+1)}\frac{p!k!}{(2k)!}, & p-k=\text{even} \\ 0, & p-k=\text{odd} \end{cases}. \tag{5}$$

3. AFFINE LEGENDRE MOMENT INVARIANTS

3.1. Affine transformation

The affine transformation can be represented by [4]

$$\begin{pmatrix} X \\ Y \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} \tag{6}$$

where $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is called the homogeneous affine transformation matrix.

The 2-D ($p+q$)th order Legendre moment of the affine transformed image $g(X, Y)$ is defined by

$$L_{p,q}^{(g)} = \det(A)\int_{-1}^{1}\int_{-1}^{1} P_p(a_{11}x+a_{12}y)P_q(a_{21}x+a_{22}y)f(x,y)dxdy, \quad (7)$$

where $\det(A)$ denotes the determinant of the matrix A.

We can associate the Legendre moments of the affine transformed image given by (7) with those of the original image. By replacing the variable x by $a_{11}x+a_{12}y$ in (2), we have

$$\begin{aligned} P_p(a_{11}x+a_{12}y) &= \sum_{m=0}^{p} c_{p,m}(a_{11}x+a_{12}y)^m \\ &= \sum_{m=0}^{p}\sum_{s=0}^{m}\binom{m}{s} c_{p,m} a_{11}^s a_{12}^{m-s} x^s y^{m-s}. \end{aligned} \quad (8)$$

Similarly,

$$P_q(a_{21}x+a_{22}y) = \sum_{n=0}^{q}\sum_{t=0}^{n}\binom{n}{t} c_{q,n} a_{21}^t a_{22}^{n-t} x^t y^{n-t}. \quad (9)$$

Substituting (8) and (9) into (7) yields

$$\begin{aligned} L_{p,q}^{(g)} &= \det(A)\int_{-1}^{1}\int_{-1}^{1}\sum_{m=0}^{p}\sum_{s=0}^{m}\sum_{n=0}^{q}\sum_{t=0}^{n}\binom{m}{s}\binom{n}{t} c_{p,m}c_{q,n} \\ &\quad \times a_{11}^s a_{12}^{m-s} a_{21}^t a_{22}^{n-t} x^{s+t} y^{m+n-s-t} f(x,y)dxdy. \end{aligned} \quad (10)$$

Using (4), we have

$$x^{s+t} = \sum_{i=0}^{s+t} d_{s+t,i}P_i(x), \qquad y^{m+n-s-t} = \sum_{j=0}^{m+n-s-t} d_{m+n-s-t,j}P_j(y). \quad (11)$$

Substitution of (11) into (10) leads to

$$\begin{aligned} L_{pq}^{(g)} &= \det(A)\sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{t=0}^{n}\sum_{i=0}^{s+t}\sum_{j=0}^{m+n-s-t}\binom{m}{s}\binom{n}{t}(a_{11})^s \\ &\quad \times (a_{12})^{m-s}(a_{21})^t(a_{22})^{n-t} c_{p,m}c_{q,n}d_{s+t,i}d_{m+n-s-t,j}L_{ij}^{(f)}. \end{aligned} \quad (12)$$

Equation (12) shows that one Legendre moment of the transformed image is a linear combination of the original image.

3.2. Affine Legendre moment invariants

The XSR decomposition introduced by Rothe [4] decomposes the affine matrix A into an x-shearing, an anisotropic scaling and a rotation matrix as follows

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix}\begin{bmatrix} \alpha_0 & 0 \\ 0 & \delta_0 \end{bmatrix}\begin{bmatrix} 1 & \beta_0 \\ 0 & 1 \end{bmatrix}, \quad (13)$$

where the coefficients α_0, δ_0 and β_0 are real numbers, and θ_0 is the rotation angle between 0 and 2π.

Based on this decomposition and using (12), we can derive a set of Legendre moment invariants I_{pq}^{xsh}, I_{pq}^{as} and I_{pq}^{ro} which are invariant to x-shearing, anisotropic scaling and rotation respectively.

Theorem 1. Let f be an original image and g its x-shearing transformed version such as $g(X, Y) = f(x+\beta_0 y, y)$. Then the following $I_{pq}^{xsh(f)}$ are invariant to x-shearing

$$I_{pq}^{xsh(f)} = \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{i=0}^{s}\sum_{j=0}^{m+n-s}\binom{m}{s}\beta_f^{m-s}c_{p,m}c_{q,n}d_{s,i}d_{m+n-s,j}L_{ij}^{(f)}, \quad (14)$$

where β_f is a parameter associated with the image f such that $\beta_f=\beta_g+\beta_0$.

Theorem 2. Let f and g be two images of the same shape but distinct scale, i.e. $g(X, Y) = f(\alpha_0 x, \delta_0 y)$. Then I_{pq}^{as} is invariant to anisotropic scaling

$$I_{pq}^{as(f)} = \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{i=0}^{m}\sum_{j=0}^{n}\alpha_f^{m+1}\delta_f^{n+1}c_{p,m}c_{q,n}d_{m,i}d_{n,j}L_{ij}^{(f)}, \quad (15)$$

where α_f and δ_f are two parameters associated with the image f such that $\alpha_f = \alpha_0\alpha_g$ and $\delta_f = \delta_0\delta_g$.

Theorem 3. Suppose the image $f(x, y)$ is rotated with an angle θ_0 to a transformed image g, $g(X, Y) = f(x\cos\theta_0+y\sin\theta_0, -x\sin\theta_0+y\cos\theta_0)$, then the rotation invariants of Legendre moments are given by

$$\begin{aligned} I_{pq}^{ro(f)} &= \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{t=0}^{n}\sum_{i=0}^{s+t}\sum_{j=0}^{m+n-s-t}\binom{m}{s}\binom{n}{t}(-1)^t \\ &\quad \times(\cos\theta_f)^{n+s-t}(\sin\theta_f)^{m+t-s}c_{p,m}c_{q,n}d_{s+t,i}d_{m+n-s-t,j}L_{ij}^{(f)} \end{aligned} \quad (16)$$

where θ_f is a parameter associated with the image f such that $\theta_f=\theta_g+\theta_0$.

The proof of Theorem 1-3 is given in Appendix.

A linear combination of (14), (15) and (16) that are respectively invariant to x-shearing, anisotropic scaling and rotation results in a set of affine Legendre moment invariants (ALMIs).

As described above, the parameters β_f, α_f, δ_f and θ_f in (14)-(16) are image dependant. An approach to determine these parameters is introduced below.

From (14), we have

$$I_{11}^{xsh(f)} = \beta_f(L_{00}^{(f)} + 2L_{02}^{(f)}/\sqrt{5}) + L_{11}^{(f)}. \quad (17)$$

Letting $I_{11}^{xsh(f)} = 0$, we obtain

$$\beta_f = -\frac{L_{11}^{(f)}}{L_{00}^{(f)} + 2L_{02}^{(f)}/\sqrt{5}}. \quad (18)$$

Setting $I_{20}^{as(f)} = I_{02}^{as(f)} = 1$ in (15), we have

$$\alpha_f = \sqrt{\frac{\sqrt{b_f^2+4a_f}-b_f}{2a_f}}, \quad \delta_f = \sqrt{\frac{\sqrt{b_f'^2+4a_f'}-b_f'}{2a_f'}}, \quad (19)$$

where

$$\begin{aligned} a_f &= \sqrt{\frac{V_f^3}{U_f}}, b_f = -\frac{\sqrt{5}}{2}L_{00}^{(f)}\sqrt{\frac{V_f}{U_f}}, a_f' = \sqrt{\frac{U_f^3}{V_f}}, b_f' = -\frac{\sqrt{5}}{2}L_{00}^{(f)}\sqrt{\frac{U_f}{V_f}}, \\ U_f &= L_{02}^{(f)} + \frac{\sqrt{5}}{2}L_{00}^{(f)}, \quad V_f = L_{20}^{(f)} + \frac{\sqrt{5}}{2}L_{00}^{(f)}. \end{aligned} \quad (20)$$

and for (16)

$$\theta_f = \arctan\left(-\left(\frac{3\sqrt{105}}{35}L_{30}^{(f)} + L_{12}^{(f)}\right)\Big/\left(\frac{3\sqrt{105}}{35}L_{03}^{(f)} + L_{21}^{(f)}\right)\right) \quad (21)$$

The parameters β_g, α_g, δ_g and θ_g associated with the transformed image $g(X, Y)$ can also be estimated according to (18), (19) and (21). It can be verified that the parameters provided by the above method satisfy the following relationships: $\beta_f=\beta_g+\beta_0$, $\alpha_f = \alpha_0\alpha_g$, $\delta_f = \delta_0\delta_g$ and $\theta_f = \theta_g+\theta_0$ where α_0, δ_0, γ_0 and β_0 are the coefficients of the affine transform applied to f.

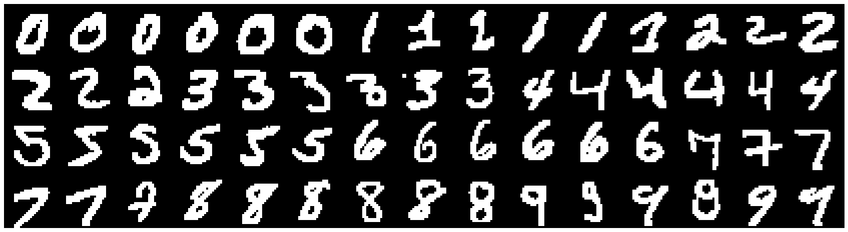

Fig 1. Some examples of MINST database.

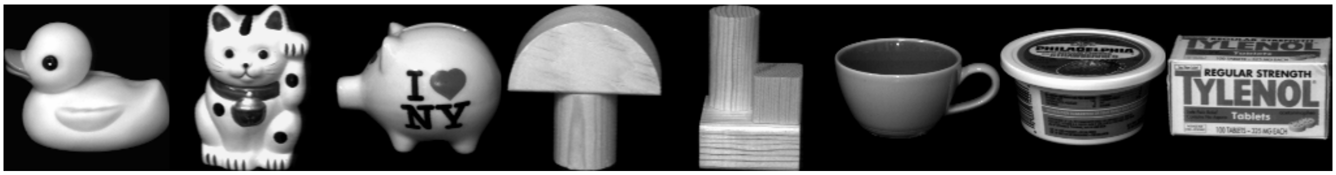

Fig.2. Eight objects chosen from Coil-100 image database.

4. EXPERIMENTAL RESULTS

Table 1. The recognition rates of different methods in digital recognition

Num	[2]	[4]	ALMIs
0	100%	100%	100%
1	63.79%	75%	99.59%
2	100%	100%	100%
3	100%	100%	100%
4	100%	100%	100%
5	87.86%	88.68%	93.83%
6	87.04%	88.07%	98.77%
7	70.99%	77.98%	100%
8	99.38%	99.38%	100%
9	100%	100%	100%

The first experiment is carried out to demonstrate the discrimination power of Reiss's method [2], Rothe's method [4] and ALMIs using a set of 0-9 real handwriting digitals chosen from the well-known MINST database [9]. Each MINST image has been size-normalized and centered in a 28×28 box. Due to large memory requirements and computational cost, we select 60 testing images from the MINST database for this experiment: 10 images for each digit (shown in Fig. 1). The testing set is generated by applying an affine transformation to the original image set with a_{11} = 1, 1.1, 1.2; a_{12} = 0, 0.1, 0.2; a_{21} = 0, 0.1, 0.2; a_{22} = 1, 1.1, 1.2; forming a set of 4860 images. Four ALMIs I_{30}, I_{21}, I_{12}, I_{03} are used as the extraction features for the system input. The same four invariants order up to 3 of other two methods are used for equal comparison. The Euclidean distance nearest neighbor classifier is used to ascertain classification accuracy. The recognition rates of the testing images for each number using the different methods is indicated in Table 1. It is evident that ALMIs achieve the best performance when compared to the other two methods.

Table 2. The recognition rates of different methods in object recognition

		[2]	[4]	ALMIs
Noise-free		90%	95%	100%
Multi-plicative noise	density=0.01	83.2%	84.44%	98.37%
	density=0.02	77.99%	80.27%	97.2%
	density=0.03	72.92%	75.26%	94.6%
Gaussian noise	STD=1	83.59%	84.64%	100%
	STD=2	79.62%	80.92%	97.85%
	STD=3	70.51%	71.81%	89.71%
Salt-and-Pepper noise	density=0.4%	81.51%	82.36%	93.16%
	density=0.8%	75.59%	75.65%	87.04%
	density=1.2%	71.16%	71.16%	82.75%
Computation time		39.03s	466.42s	154.63s

In the second experiment, we test the classification behavior of the proposed affine Legendre moment invariants under various image noises. An original set of eight objects are selected from the Coil-100 image database of Columbia University ([10], see Fig. 2). The testing set is generated by applying an affine transformation to the original image set with a_{11} = 1, 1.02, 1.04, 1.06; a_{12} = 0, 0.01, 0.02, 0.03; a_{21} = 0, 0.02, 0.04, 0.06; a_{22} = 1, 1.01, 1.03, 1.05; forming a set of 2048 images. This is followed by adding multiplicative noise with different noise densities, white Gaussian noise with different standard deviations (STD) and salt-and-pepper noise with different noise densities. The number, order of invariants and the classifiers are the same as the first experiment. Table 2 shows the classification results using different moment invariants. One can observe from this table that high recognition results are obtained for different methods in the noise-free case. Note that the recognition accuracy decreases with an increase in noise level. However, ALMIs perform better than affine geometric moment invariants by direct method [2] and by image normalization method [4] in terms of recognition for noisy images, irrespective of the type of noise.

We also compared the computational speed of [2], [4] and ALMIs in the above experiment. The computation time required for one recognition task in [2], [4] and ALMIs are respectively 39.03, 466.42 and 154.63 seconds as shown in Table 2. Note that the program was implemented in MATLAB 7 on a PC Dual-Core T4400 2.2 GHZ, 2G RAM. It can be seen that in [2] computation is much faster than [4] and ALMIs. This is due to the fact that [2] can be easily and explicitly expressed by central moments.

5. CONCLUSION

In this paper, we proposed a new approach to derive a set of affine invariants using the orthogonal Legendre moments. The major theoretical contribution of this work concerns Theorems 1-3 and Equation (17)-(21). It proves that the affine Legendre moment invariants can be expressed as a linear combination of the *x*-shearing, anisotropic scaling and rotation invariants of Legendre moments. The experiments conducted under varying noise and noise free configurations demonstrate the discriminative power of the proposed descriptors, which clearly outperform existing methods.

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7. APPENDIX

Proof of Theorem 1: Let $g(X, Y) = f(x+\beta_0 y, y)$, then the *x*-shearing Legendre moment invariants of the image intensity function $g(X, Y)$ is defined as

$$\begin{aligned} I_{pq}^{xsh(g)} &= \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{i=0}^{s}\sum_{j=0}^{m+n-s}\binom{m}{s}\beta_g^{m-s}c_{p,m}c_{q,n}d_{s,i}d_{m+n-s,j}L_{ij}^{(g)} \\ &= \int_{-1}^{1}\int_{-1}^{1}P_p(X+\beta_g Y)P_q(Y)g(X,Y)dXdY \end{aligned} \quad \text{(A1)}$$

Using $\beta_f=\beta_g+\beta_0$, we have

$$\begin{aligned} &P_p(X+\beta_g Y) = P_p(x+\beta_f y), \\ &P_q(Y) = P_q(y), \end{aligned} \quad \text{(A2)}$$

Substituting (A2) into (A1) yields

$$I_{pq}^{xsh(g)} = I_{pq}^{xsh(f)}.$$

Proof of Theorem 2: Let $g(X, Y) = f(\alpha_0 x, \delta_0 y)$, then the scale Legendre moment invariants of the image intensity function $g(X, Y)$ is defined as

$$\begin{aligned} I_{pq}^{as(g)} &= \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{i=0}^{m}\sum_{j=0}^{n}\alpha_g^{m+1}\delta_g^{n+1}c_{p,m}c_{q,n}d_{m,i}d_{n,j}L_{ij}^{(g)}, \\ &= \alpha_g\delta_g\int_{-1}^{1}\int_{-1}^{1}P_p(\alpha_g X)P_q(\delta_g Y)g(X,Y)dXdY \end{aligned} \quad \text{(A3)}$$

With the help of $\alpha_f = \alpha_0\alpha_g$ and $\delta_f = \delta_0\delta_g$, we have

$$\begin{aligned} &P_p(\alpha_g X) = P_p(\alpha_f x), \\ &P_q(\delta_g Y) = P_q(\delta_f y), \end{aligned} \quad \text{(A4)}$$

Substituting (A4) into (A3) yields

$$I_{pq}^{as(g)} = I_{pq}^{as(f)}.$$

Proof of Theorem 3: Let $g(X, Y) = f(x\cos\theta_0+y\sin\theta_0, -x\sin\theta_0+y\cos\theta_0)$, the rotation Legendre moment invariants of the image intensity function $g(X, Y)$ is defined as

$$\begin{aligned} I_{pq}^{ro(g)} &= \sum_{m=0}^{p}\sum_{n=0}^{q}\sum_{s=0}^{m}\sum_{t=0}^{n}\sum_{i=0}^{s+t}\sum_{j=0}^{m+n-s-t}\binom{m}{s}\binom{n}{t}(-1)^t(\cos\theta_g)^{n+s-t} \\ &\quad \times(\sin\theta_g)^{m+t-s}c_{p,m}c_{q,n}d_{s+t,i}d_{m+n-s-t,j}L_{ij}^{(g)} \\ &= \int_{-1}^{1}\int_{-1}^{1}P_p(\cos\theta_g X+\sin\theta_g Y) \\ &\quad \times P_q(-\sin\theta_g X+\cos\theta_g Y)g(X,Y)dXdY \end{aligned} \quad \text{(A5)}$$

With the help of triangle formula and $\theta_f - \theta_g = \theta_0$, we have

$$\begin{aligned} &P_p(\cos\theta_g X+\sin\theta_g Y) \\ &= P_p\left[\cos\theta_g(x\cos\theta_0+y\sin\theta_0)+\sin\theta_g(-x\sin\theta_0+y\cos\theta_0)\right] \\ &= P_p(\cos\theta_f x+\sin\theta_f y) \end{aligned} \quad \text{(A6)}$$

Similarly, we have

$$P_q(-\sin\theta_g X+\cos\theta_g Y) = P_q(-\sin\theta_f x+\cos\theta_f y) \quad \text{(A7)}$$

Substituting (A6) and (A7) into (A5) yields

$$I_{pq}^{ro(g)} = I_{pq}^{ro(f)}.$$