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The Effect of Fuzzy Resolution in Servo Tuning

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Abstract

Optimization of knowledge-based servo tuning requires consideration of several issues including the selection of the fuzzy resolution, the shape of the membership functions of fuzzy variables, and the level of overlapping between different fuzzy sets. This paper studies the effect of fuzzy resolutions on the processing speed, storage requirement, and response accuracy of a hierarchical fuzzy tuning system. First, some analytical results are given, and next experimental examples are presented. The responses from the experimental system with different fuzzy resolution are compared.

1. Introduction

Fuzzy control has been found effective in many applications, especially in situations where vagueness, subjectivity, and uncertainties are involved. The basic idea of fuzzy control is to make use of the knowledge and experience from experts to form a rule-base with linguistic *if-then* rules. Proper control actions are then derived from the rule-base which can be considered as an emulation of the behaviour of human operators. Typically, a fuzzy controller is placed in the direct control loop and used to replace a conventional, hard (crisp) control algorithm.

An alternative approach was proposed by de Silva and Macfarlane [1]. They considered a hierarchical control structure in that all the direct control algorithms are placed at a lower level, and a knowledge-based system for tuning the direct controller is placed at an upper level. It was suggested that this structure naturally conforms to the fact that experience-based tuning protocols are inherently fuzzy in general. Furthermore, in this structure, fuzzy schemes are relegated to a higher level so that they do not enter into the direct control loops, thereby maintaining the control bandwidth high.

Although the knowledge-based algorithm is placed at a different level in the control system with a fuzzy tuner, its design procedure is the same as in the case of fuzzy control. A typical design procedure for a fuzzy controller may be found in [2]. An important step in that design procedure is the determination of fuzzy resolution and, once the fuzzy resolution is determined, of the fuzzy partitions of the input and output spaces. ***Fuzzy resolution*** is defined as the total number of fuzzy states, within a given knowledge base, for a fuzzy variable [3].

Generally speaking, the higher the fuzzy resolution that the variables can assume, the more accurate the control a system can achieve. However, the higher resolution will generally increase the computational time and consequently may affect the performance of the control system or require more expensive hardware. This paper will study these important issues, both analytically and through experimental examples.

The organisation of the paper is as follows: Some basic concepts related to fuzzy resolution are introduced in Section 2. The effect of fuzzy resolution on processing time, accuracy, and stability is studied in Section 3. Section 4 presents several experimental studies to demonstrate the effect of fuzzy resolution in a practical system. Finally, concluding remarks are given in Section 5.

2. The Concept of Fuzzy Resolution

Theoretically, a fuzzy variable may assume any finite number of fuzzy states (When there is an infinite number of fuzzy states, a fuzzy variable becomes a crisp variable.). In practice, however, a fuzzy variable typically assumes $3 \sim 9$ fuzzy states. For example, when an operator describes a control action, he may use negative large (NL), negative small (NS), no change (NC), positive small (PS), and positive large (PL). Therefore, fuzzy resolution represents an approximate characterization level to the state of a variable. To be precise, we shall in the following sections define fuzzy resolution of a variable through its membership functions.

Here we only look at a few specific situations which are often encountered in practical applications. We assume that each fuzzy state of the variable will have a unique modal point (typically, having a peak membership grade) in the membership function, then the number of such modal points in the membership functions (within its support set) corresponds to the number of fuzzy states of the variable (within the knowledge base). Let us further assume that the membership functions are symmetrical and they uniformly partition the support set. Then the intermodal spacing of the membership may be used as a measure of the fuzzy resolution of the variable. Mathematically, this can be described as follows.

Let X be a universe of discourse, x be its generic element, and $\mu_A(x)$: $\mathbf{R} \rightarrow [0,\ 1]$ be the membership function of a fuzzy set A. Then an appropriate quantitative definition for fuzzy resolution would be

$$r_f = \frac{w_s}{w_m} \tag{1}$$

where w_s represents the width of the support set x of A, and w_m is the intermodal spacing. If the membership functions have trapezoidal shapes, the intercentroidal spacing (the distance between the centroids of two fuzzy sets) should be used in place of the intermodal spacing.

Fuzzy resolution also represents a measure of ***fuzziness***. The degree of fuzziness is assumed to express, on a global level, the difficulty of deciding which elements belong and which do not belong to a given fuzzy set [4]. There are several quantitative measures available in the literature [4,5]. In [3], the measure of the degree of fuzziness of a fuzzy set A is defined with respect to a fuzzy state or a modal point in A, and expressed as

$$d_f = \frac{1}{m}\int_X f(x)dx \tag{2}$$

in which the nonnegative function $f(x)$ is defined in the support set X of the fuzzy set A by $f(x)$: $X \rightarrow [0, 0.5]$ with

$$f(x) = \begin{cases} \mu_A(x) & \text{for } \mu_A(x) \leq 0.5 \\ 1 - \mu_A(x) & \text{otherwise} \end{cases} \tag{3}$$

and m is the number of modal points in A. With this definition, generally higher the fuzzy resolution, lower the degree of fuzziness.

In servo tuning, if the tuning knowledge is limited, a tuning system with a low fuzzy resolution may be implemented; if there exists a substantial amount of tuning knowledge, a fuzzy system with high resolution is possible. In the extreme case, if crisp knowledge is available, we say that the system has an infinitesimal resolution. The effect of fuzzy resolution on such factors as processing speed, storage requirement, and accuracy will be analysed in detail in the next section.

3. The Effect of Fuzzy Resolution

To analyse the effect of fuzzy resolution in servo tuning, we need to outline the major steps involved in designing the knowledge-based tuner. Figure 1 presents the general structure of a knowledge-based, hierarchical, servo tuning system [6]. As can be seen from the figure, the overall tuner consists of a three level hierarchy: the direct control level, the servo expert level and knowledge-based fuzzy tuning level. In the tuning mode of opera-

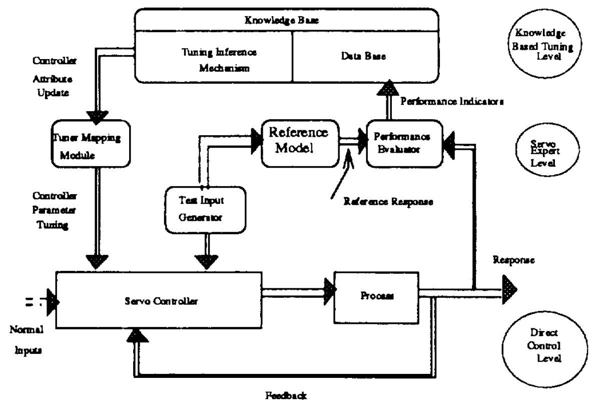

Figure 1: **The General Structure of a Fuzzy Tuning System.**

tion, the same test input is applied to both the process and a reference model. The two responses obtained in this manner are compared by the performance evaluator at the servo-expert level. Thus the degree of agreement of the two responses is determined and used by the knowledge-based tuning level. The upper level then uses this information to update the controller attributes by using compositional rule of inference and the centroidal defuzzification method. These attributes are then converted into new controller parameters by a tuner mapping module. Since the effect of fuzzy resolution comes from the knowledge-based part of the system, we shall concentrate our analysis on the part starting from the measurement of the condition variables to the derivation of tuning the variables of tuning action.

To begin our analytical study, some notations are introduced. Consider a rule base that is given by the general form

$$\underset{i,j,\ldots,k}{\textbf{Else}} \; [\textbf{If } e_1 \text{ is } E_1^i \text{ and } \ldots \text{ and } e_n \text{ is } E_n^a \textbf{ then } c_1 \text{ is } C_1^j \text{ and } \ldots \text{ and } c_p \text{ is } C_p^b] \tag{4}$$

Here, $[e_1, \ldots, e_n]^T$ (denoted by $\underline{e}$) $\in \mathbf{R}^n$ is a vector of condition variables which are monitored for subsequent control. e_s, $s = 1, ..., n$, may assume one of m discrete fuzzy states $E_s^1, E_s^2, ..., E_s^m$ (e.g., in specification (IN), poor(PR), and very poor (VP)). $[c_1, ..., c_p]^T$ (denoted by $\underline{c}$) is a vector of action variables which are subsequently converted to tuning-parameter values. c_q, $q = 1, ..., p$, assumes one of r fuzzy states $C_q^1, C_q^2, ..., C_q^r$ (e.g., NL, NS, NC, PS, and PL). E and C denote the fuzzy variables of error condition and tuning action respectively. The attached subscript and superscript respectively identify a particular variable in a vector of variables and a particular fuzzy state of the variable. It should be noted that m and r are measures of the fuzzy resolution of e_s and c_q, respectively. The operations to obtain $\underline{c}$ from $\underline{e}$ are outlined next.

Fuzzification

The operations involved starting from the step of reading the measurements of the condition variables $\underline{e}$ to the generation of fuzzy variables E is a *fuzzification* process, e.g., e_s is matched to its corresponding fuzzy variable by finding the matching membership value, such as:

$$\mu_{E_s}(e_s) : \mathbf{R} \rightarrow [0, 1].$$

Compositional Rule of Inference

The membership function of the rule base represented by equation (4) can be expressed as

$$\mu_R\ (\underline{e},\ \underline{c})\ =\ \sup_{\underline{i},\underline{j}}\ \min_{(\underline{i})}\ \{\mu_{\underline{E}^{\underline{i}}},\ \mu_{\underline{C}^{\underline{i}}}\ (\underline{c})\} \qquad (5)$$

where

$$\mu_{\underline{E}^{\underline{i}}} \triangleq \{\mu_{E_1^i}(e_1), ..., \mu_{E_n^i}(e_n)\}$$

$$\mu_{\underline{C}^{\underline{i}}}(\underline{c}) \triangleq \{\mu_{C_1^j}(c_1), ..., \mu_{C_p^j}(c_p)\}$$

$$\mu_R(\underline{e}, \underline{c}) : \mathbf{R}^n \times \mathbf{R}^p \rightarrow [0, 1].$$

Thus, using the *compositional rule of inference*, $\mu_{\underline{C}^{\underline{i}}}(\underline{c})$ can be obtained as follows:

$$\mu_{\underline{C}^{\underline{i}}}\ (\underline{c})\ =\ \sup_{\underline{e}}\ \min\ \{\ \mu_{\underline{E}^{\underline{i}}}\ (\underline{e}),\ \mu_R(\underline{e}, \underline{c})\} \qquad (6)$$

Defuzzification

Fuzzy action variables obtained as described in the previous section, have to be defuzzified into crisp quantities. This can be accomplished by the centroidal defuzzification method

$$\hat{c}_q = \frac{c_q \int c\mu_{C_q}(c)dc}{c_q \int \mu_{C_q}(c)dc} \quad \text{for } q = 1, 2, ..., p \qquad (7)$$

where $\hat{c}_q$ is a vector of real valued outputs which can be subsequently converted into crisp quantities of control parameters.

Having outlined the major computational steps involved in finding the values of tuning action variables from the measurement of condition variables, we can now proceed to an investigation of the effect of the fuzzy resolution on the tuning system.

Processing Speed

If Equation (5) is used for computing the rule base and Equation (6) is used for inference, the total number of "min"s and "sup"s required to compute $\mu_{\underline{C}^{\underline{i}}}(\underline{C})$ from $\mu_{\underline{E}^{\underline{i}}}$ are [6]:

$$\text{Total number of "min"s} = ((n + p - 1)m^n N^{n+p} + pnN^{n+1})\text{"min"s, and} \qquad (8)$$

$$\text{Total number of "sup"s} = ((m^n - 1)N^{n+p} + pN(N^n - 1))\text{"sup"s,} \qquad (9)$$

where N represents the number of discrete points used to digitally represent any of the membership functions $\mu_{E_s}(e)$ and $\mu_{C_q}(c)$. As can be seen from Equations (8) and (9), the computing time is significantly affected by the fuzzy resolution m of e_s. But the fuzzy resolution r of c_q does not appear in the equations. This is because: (1) the maximum number of possible rules (m^n) in the rule base are determined by the number of condition variables and their fuzzy resolution m, hence the computation of the rule base is not affected by the fuzzy resolution r of c_q; (2) the computation of the inference concerns only the rule base and the membership functions of $\underline{e}$, i.e., the fuzzy resolution r does not directly participate in the inference procedure. Consequently, fuzzy resolution of c_q does not affect the computation of the inference. However, fuzzy resolution of c_q does affect the defuzzification process. But, compared to the time for computing the rule base and the inference, the time taken for the fuzzification and defuzzification processes is negligible. Accordingly, the analysis for this computation is omited here.

To graphically illustrate how the fuzzy resolution m affects the total computing time, the relations represented by the Equations (8) and (9) are shown in Figure 1. In the figure, we assume that N=100, n=5, p=4, and m varies from 1 to 15. We also assume that it takes the same computing time unit for "min" and "sup" operations. As can be seen from the figure, the computing time increases exponentially with the fuzzy resolution m of e_s.

Storage Requirement

Again assume that N is the number of discrete points used to digitally represent any of the membership functions. Then, the number of memory units required to store the membership functions for the condition vari-

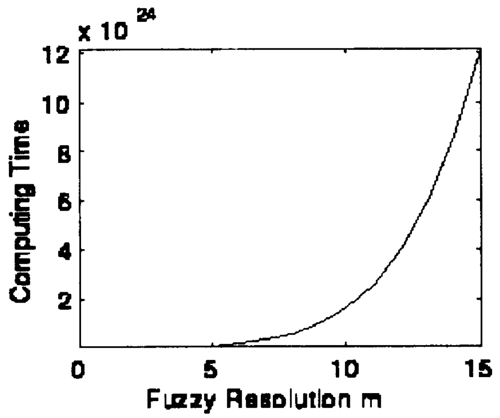

Figure 2: **Illustration of Computation Time vs. Fuzzy Resolution m.**

ables and the action variables is mnN and rpN, respectively. The number of memory units needed for storing the rule base membership functions (the fuzzy relation matrix) is np. This shows that the effect of the fuzzy resolution m and r is not very significant to the storage requirement, mainly because the problem itself does not need substantial memory space. In practice, more memory space is required since a large number of temporary variables has to be used during the computation process. However, as the exact amount of memory space used for this purpose is very much dependent on programming skills, this is not discussed here.

Accuracy

If a low fuzzy resolution is used in the system, the number of rules that define different input conditions is limited, and there is a good possibility that no rule exists for certain inputs. Such undefined situations lower the efficiency of the fuzzy controller. This problem can be avoided or partially resolved by increasing the fuzzy resolution, and consequently the accuracy of the system can be improved.

Furthermore, only a coarse tuning action may be generated by a system with low fuzzy resolution. Hence, it is possible that the system will exhibit a hunting response about the desired position. On the other hand, as mentioned earlier, the processing speed is drastically affected by the fuzzy resolution, and accordingly, the heavy computational burden resulting from a high fuzzy resolution can slow down the response. In practical implementations, this could be avoided by developing a crisp decision table off-line and using this table in a table look-up mode to determine a crisp control action during operation.

4. Experiments on Servo Tuning

The experiments are conducted on a DMC400 servo motor system. The DMC400 is a PC bus compatible, programmable motion controller for DC motors, developed by Galil Motion Control, Inc. Figure 3 is a block diagram representation of the hardware and software system. The left-hand block represents the hardware of the system; specifically, the motor, encoder, amplifier, power supply and the controller. The right-hand block represents the software part of the system. It is arranged in three levels as shown in Figure 1. Communication programs interface these two blocks. Particularly, the measured position of the servo-motor is transferred to the subroutines in the servo-expert level, and the updated controller parameters are returned from that level to the hard controller. To test the effect of

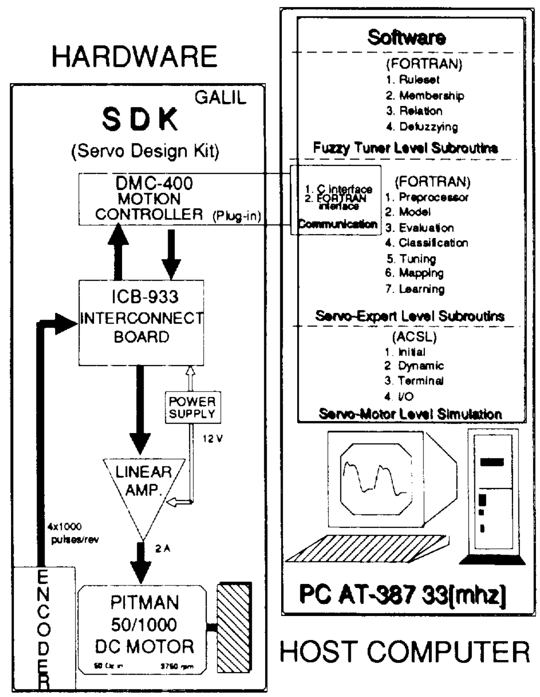

Figure 3: **The Experimental System.**

fuzzy resolution on the response of the tuning system, two schemes are implemented, reflecting different levels of fuzzy resolution. The first one directly implements the rule base shown by Equation (3). In this approach, every time the fuzzy resolutions change, the rule base has to be modified and the entire problem has to be reformulated following the procedures of fuzzification, application of the compositional rule of inference, and defuzzification.

The second scheme uses the so-called incremental rule base [6], which is derived by using the concepts of rule disassociation and fuzzy resolution, in particular the resolution relationships. In this approach, the fuzzy resolution is lined with an index i and can be refined arbitrarily and varied during operation of the tuner, without having to reformulate the problem.

The latter approach is certainly advantageous in terms of the convenience in analyzing and implementing the knowledge-based tuner, especially when studying the effect of the fuzzy resolutions of the condition and action variables. However, since the experiments conducted in this paper were based on some of the existing software reported in [7], the first approach is still used here.

During the on-line tuning experiments, five system-response related attributes are measured. These are the 95% rise time, damped natural frequency, damping ratio, overshoot, and offset. The measured absolute quantities of these variables are then converted into relative measures with respect to the values derived from a desired response. The relative measures thus derived are used as condition variables, denoted as c_1, c_2, c_3, c_4, and c_5, respectively. Four controller-parameter-related attributes used are the action variables. These are compensator phase lead at regular crossover frequency, regular crossover frequency, compensator magnitude at cross over frequency, and low frequency of integrator for steady-state accuracy, denoted as ϕ_m, $\bar{\omega}_c$, g_c, and $\bar{\omega}_l$, respectively, for the convenience of later analysis.

The action variables are calculated using both $\underline{c}$ and the decision table which is computed offline. Usually different fuzzy resolutions m and r result in different rules, and consequently different decision tables. The detailed procedure for generating decision tables may be found in [2]. To show how fuzzy resolutions affect the response of practical systems, four experiments are conducted. A fuzzy resolutions of 3, 5, and 7, respectively, are used in the first three experiments in forming the knowledge base and computing the decision table. In the fourth experiment, analytical relations are introduced to replace some of the rules in the rule base, for tuning the system.

Figure 4(a), (b), and (c) respectively illustrate the system responses obtained from the first three experiments, respectively. Since the decision table is generated off-line, no processing speed problem is involved here. So, all what matters is the time required to settle the response, or in other words, at a given time, what is the accuracy of the response. The figures show that for the response to reach the given accuracy limit (which is the same for all experiments), the system with a higher fuzzy resolution takes less time. This is also consistent with the analysis given in Section 3. It should also be noted that the speed-up of the settling time as the fuzzy resolution is increased from 3 to 5, is dramatic, and the speed-up as the fuzzy resolution increase from 5 to 7 is relatively low.

Figure 4(d) shows the response of the system tuned using a set of analytical relations. During the experiment, it was found that it is very difficult to obtain four analytical relations to tune ϕ_m, $\bar{\omega}_c$, g_c, and $\bar{\omega}_l$ simultaniously. So instead, two crisp analytical relations are derived to tune g_c and $\bar{\omega}_l$, and the decision table is still used to tune ϕ_m and $\bar{\omega}_c$. The relations for updating ϕ_m, $\bar{\omega}_c$, g_c, and $\bar{\omega}_l$ are,

$$\begin{aligned}
\phi_m(t_{k+1}) &= \phi_m(t_k) + \Phi_m \Gamma_{\phi_m}(\underline{c}), \\
\bar{\omega}_c(t_{k+1}) &= \bar{\omega}_c(t_k) + \bar{\Omega}_c \Gamma_{\bar{\omega}_c}(\underline{c}), \\
g_c(t_{k+1}) &= g_c(t_k) + G_c(0.5 - \frac{c_1 + c_2 + c_3 + c_4 + c_5}{0.8}), \\
\text{and} & \\
\bar{\omega}_l(t_{k+1}) &= \bar{\omega}_l(t_k) + \bar{\Omega}_l(1 - \frac{c_5}{0.4}),
\end{aligned}$$

respectively, where Φ_m, $\bar{\Omega}_c$, G_c, and $\bar{\Omega}_l$ are corresponding to the maximum incremental changes of controller attributes ϕ_m, $\bar{\omega}_c$, g_c, and $\bar{\omega}_l$. Also t_k and t_{k+1} represent the previous and the current time steps, respectively, and $\Gamma\phi_m$ and $\Gamma_{om\imath ga_c}$ are particular entries of the decision table.

As can be seen from Figure 4(d), the system performed less satisfactorily compared to the responses obtained by using fuzzy tuning which has a finite resolution (e.g. m=r=5 and m=r=7). The reason for this is that the analytical equations can only represent some of the knowledge about how to tune a system. So it is like an incomplete knowledge base and it can be quite possible that this knowledge base is less complete than the knowledge base represented by a heuristic one. It should be pointed out that the analytical relations in this case, thought it is crisp, do not represent an infinitesimal resolution case. In principle, if all the knowledge about tuning the system is completely and precisely known, that is, in the case of infinitesimal resolution, the tuning system could settle within a very short time.

5. Conclusion

In this paper, the effect of fuzzy resolutions on parameters related to system responses has been studied, both analytically and experimentally. Generally speaking, finer the resolution, higher the response accuracy, or in other words, faster the response settling time. However, a higher resolution usually means a heavier computational load. Moreover, when the fuzzy resolution

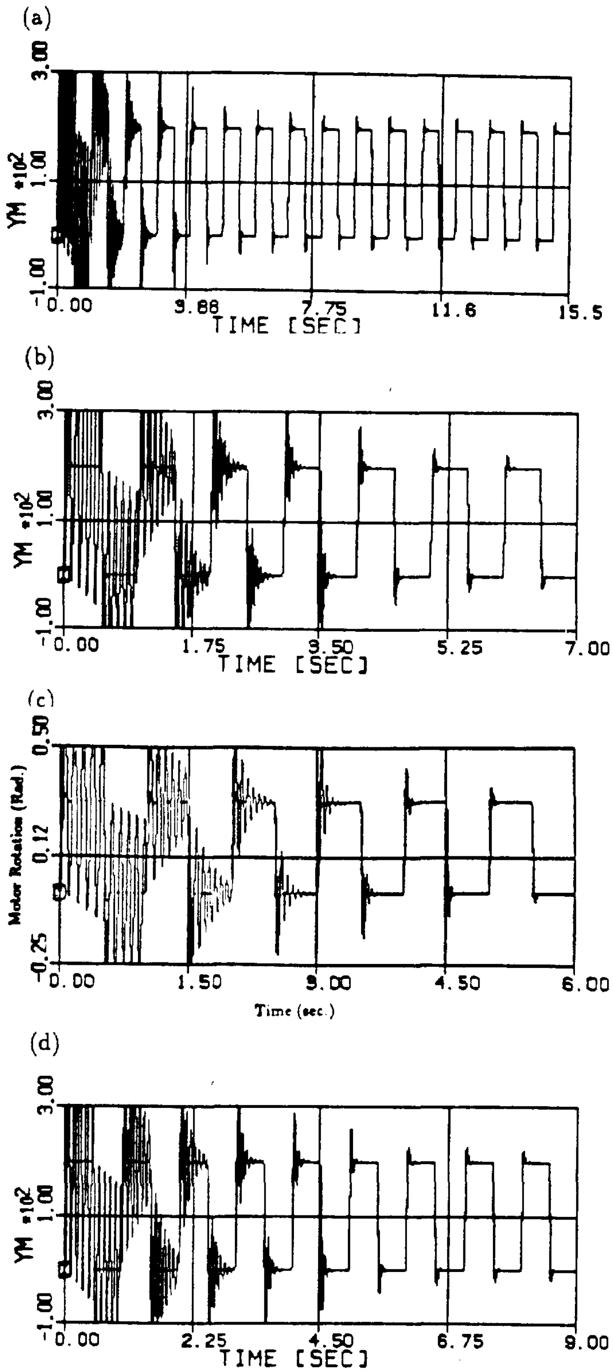

Figure 4: **Effect of Fuzzy Resolutions on System Responses. a)** $m=r=3$. **b)** $m=r=5$. **c)** $m=r=7$. **d) With Analytical Relations.**

reaches a certain level, the system response may improve only slightly (or may not improve at all) with further increase of the fuzzy resolution. Therefore, in practical applications, the fuzzy resolution has to be chosen to satisfy the requirements of the particular situation, by weighing various factors. The methods presented in this paper can be used as guidelines for further studies on the issue of the effect of fuzzy resolution. It can also be useful for the design of knowledge-based systems in control and tuning of other plants or processes.

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