Invariant Feature Set in Convex Hull for Fast Image Registration

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Abstract—**In this paper, a novel feature set in images for registration is identified. Unique, geometrically invariant and easily extractable features in images called convex diagonal, convex quadrilateral are used for accurate image registration. Convex diagonals, convex quadrilaterals have attractive properties like easy extraction, geometric invariance and frequent occurrence. Coordinates, length and orientation information of corresponding convex diagonals in different images is used for initial transformation estimate. Corresponding convex hulls of scene objects are matched using Hausdorff distance as similarity measure operator. Coarse level estimate facilitates efficient, real time computation for final registration process. Initial transformation estimate based on convex diagonals, extracted from convex hull of scene objects, is refined using fine level image details to minimize errors originating from quantization and same convex hull information for different object shapes. The behavior of reference quadrilateral is robust against noise, outliers and broken edges.**

I. Introduction

Image registration is the process of aligning two or more images, taken at different times, from different view points, and/or by different sensors. The difference in images can be due to intrinsic or extrinsic sensor parameters. Applications of image registration are environmental monitoring, image mosaicing, geographical information system, multisensor fusion, cartography, and computer vision, to name a few.

Image registration can be implemented using direct image intensity values or feature extraction and matching. But the major limitation in feature based image registration is selection of an appropriate distinctive feature, for the problem in hand, which is frequently spread and easily detectable in images. This situation becomes more complicated with varying illumination conditions, noise and occlusions in the images. Robust algorithm should be able to detect the required features in all projections and degradations with good localization accuracy.

In literature, survey on image registration algorithms can be found in [1-2]. Feature based image registration method use similarity measure operators for matching and Hausdorff distance is commonly used as a pattern matching operator [3]. Detecting different kinds of features in region, (line segments, line end points, line intersection and orientation, corners, contours, centre of gravity, area, and intensity profile) are suggested. Detection of region features through segmentation is proposed in [4]. Goshtaby et al. [5] proposed image registration with sub-pixel accuracy through iterative estimation. Yang et al. [6] suggested affine invariants using convex hull for image registration, later proved linearly dependent. Goshtaby et al. [7] proposed point pattern matching scheme using convex hull information, the suggested scheme is not appropriate for object matching due to 1) different shapes that can be represented using same convex hull 2) computationally expensive. Recently Alhichri et al. [8] suggested a new set of salient features in images – virtual circles to compute translation and scaling transformation only.

A survey of different methods for convex hull computation can be found in [9], [10], [11].

The proposed method uses unique feature set, convex diagonals and convex quadrilaterals in convex hull, to compute and validate the initial transformation estimate. The proposed method is computationally efficient and can produce real time results. This paper is organized into five parts:
1) introduction: deals with basic theory and literature survey 2) similarity measure operator: details of pattern matching technique 3) feature set in convex hull for image registration: discusses required parameters for image registration and the proposed method 4) experimental results: explains implementation of proposed method for image registration 5) conclusion and discussion: concluding remarks with discussion on advantages and limitations of the proposed method.

II. Similarity Measure Operator

Hausdorff distance measures the maximum distance of a set to the nearest point in another set [12]. Selection of distance measure is dependent on the problem in hand, as Euclidean distance can give good measure between two subset of points without consideration of shape factors.

The definition of distance between polygons can become quite unsatisfactory for some applications while the shortest distance is totally independent of each polygonal shape. Hausdorff distance takes into consideration position, shape, computing dissimilarity, while other methods are insensitive to these parameters like Manhattan distance, Euclidean distance etc. Hausdorff distance is asymmetric compared with shortest distance that exhibits symmetric property naturally. Asymmetric property of Hausdorff distance satisfies the following relationship

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$$h(A,B) \neq h(B,A)$$

where A, B are set of points. The function $h(A,B)$ is called directed Hausdorff distance from A to B. It identifies the point $a \in A$ that is farthest from any point of B and measures the distance from a to its nearest neighbor in B. The Hausdorff distance $h(A,B)$ measures the degree of mismatch between two sets. As it reflects the distance of the point of A that is farthest from any point of B and vice versa. Intuitively, if the Hausdorff distance is d then every point of A must be with in the distance d of some point of B and vice versa.

Given two sets of points $A = \{a_1, \cdots, a_p\}$ and $B = \{b_1, \cdots, b_p\}$, the Hausdorff distance is defined as [3]

$$\begin{aligned} & H(A,B) = \max(h(A,B), h(B,A)) \\ & where \\ & h(A,B) = \max_{a_i \in A} \min_{b_i \in B} p(a_i, b_i) \end{aligned} \tag{1}$$

$p(,)$ is some underlying distance function for comparing two points a_i, b_j. If sets A and B are made of lines or polygons instead of single points, then $H(A,B)$ applies to all defining points of these lines or polygons, and not only to their vertices. The brute force algorithm could no longer be used for computing Hausdorff distance between such sets, as they involve an infinite number of points [13].

III. FEATURE SET IN CONVEX HULL FOR IMAGE REGISTRATION

The proposed method deals with the estimation of transformation parameters i.e. scaling, translation and rotation.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where (S_x, S_y), (T_x, T_y), (θ) are scaling, translation and rotation parameters respectively, to establish mapping between original point (x, y) and transformed point (x', y')

$$(x', y') = f((x, y)) \tag{3}$$

where $f(.)$ represents transformation function.

For a set of points in plane, convex hull is the smallest convex object having all points. Convex hull has attractive properties i.e. uniqueness, computationally efficient, making it suitable for representation and analysis [6]. Convex hull gives simplest approximation of a shape of object by surrounding all its points as shown in figure-1 [14]. Convex hulls can also be used to approximate more complex shapes and set of points in multi-dimensional space.

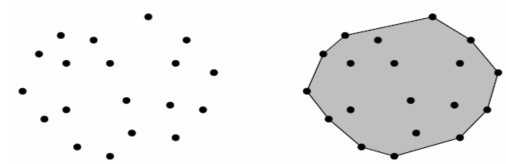


Figure. 1. Convex hull for a set of points in 2-D

Different methods have been proposed for linear time computation of convex hull. Discussion on computation of convex hull is not the main goal of current work and detailed literature review can be found in [9], [10], [11].Graham scan's algorithm with computational complexity of $O(n.\log(n))$ is used for convex hull computation in the current paper [15].

In this paper, convex diagonal information of scene objects is used for initial transformation estimate i.e. $E_i \ \forall i = \{1,2,\ldots,M\}$ where M represents total number of computed convex hulls in image. For each convex hull computed in image, a convex quadrilateral is also calculated that joins four extreme vertices into four directions of convex hull. Convex quadrilateral is used for verification process in image registration.

In remainder of paper, term reference image, $I_r(x,y)$, refers to original image while sensed image, $I_s(x,y)$, refers to transformed image. Length, orientation of convex diagonals (reference diagonal), longest diagonals in convex hulls, is used for scaling and rotation estimate while translation information is computed using coordinates of mid points of corresponding convex diagonals in $I_r(x,y)$, $I_s(x,y)$. Dissimilarity distances between shapes of convex hulls are calculated and transformation parameters are estimated. Hausdorff distance is used to measure dissimilarity among convex hulls in $I_r(x,y)$, $I_s(x,y)$.

Proposed method consists of following steps

1. *Preprocessing* – smoothing and noise filtering
2. *Computation of convex hulls* - reject nested convex hulls
3. *Compute reference/convex diagonals, reference points for individual scene objects in* $I_r(x,y)$, $I_s(x,y)$.
4. *Estimate transformation parameter*
5. *Calculate Hausdorff distance between convex hulls of* $I_r(x,y)$, $I_s(x,y)$
6. *Smoothing process* – parameters selected based on

threshold criterion

7. *Verification and correction process* – transformation parameters adjustment for local descent in dissimilarity measure

The proposed method starts with a preprocessing step to minimize noise, increase understanding and accuracy of image data. Different noise filtering operators have been devised with performance based on noise model present. Gaussian smoothing filter is used in *preprocessing* step due to its separability characteristic and efficient performance.

In the next step, individual object convex hulls are computed, using Graham scan's method, resulting $M \leq N$ where M is total number of convex hulls computed and N is total number of actual objects in scene. Matching convex hull vertices among both images is not sufficient, for correct image registration, as two different shapes can have same convex hulls. Nested convex hulls are rejected due to erroneous transformation estimates and inefficient computation. From $I_r(x,y)$, convex hull with biggest area, called as reference convex hull, is selected to be matched with all convex hulls of $I_s(x,y)$ for parameter estimation and coarse level matching. Convex diagonals are also referred as reference diagonals, in remainder of paper. L_{rn}, L_{sn} are used to represent n[th] reference diagonals in $I_r(x,y)$, $I_s(x,y)$ respectively. Ray diagram method can efficiently compute convex diagonals in linear time.

$$|L_{rn}| > |L_{rm}| \; \forall\, n \neq m, n \leq M, m \leq M$$

$$|L_{sn}| > |L_{sm}| \; \forall\, n \neq m, n \leq M, m \leq M \tag{4}$$

where $|.|$ represents magnitude of line and $|L_{rn}|, |L_{sn}|$ in (4) represent longest convex diagonals in $I_r(x,y)$, $I_s(x,y)$. Intuitively $|L_{rn}| \neq |L_{sn}|$ due to the scaling transformation. Mid point of reference diagonal, reference point, used as control point for image alignment, is geometrically invariant and easily detectable in transformations, degradations and abrupt changes. As per assumption of no shearing transformation, reference diagonals in both images can be used for scaling, translation estimate.

$$scale = \frac{|L_{rn}|}{|L_{sn}|}$$

$$(x_t \quad y_t) = Mid\ (L_{rn}) - Mid\ (L_{sn}) \tag{5}$$

where $Mid\,(.)$ calculates the mid point coordinates of argument line. The rotation parameter can be calculated as

$$\theta_1 = \frac{(\alpha_2 \beta_1 - \beta_2 \alpha_1)}{(\beta_2 - \alpha_2)}$$

$$\theta_2 = \frac{(\beta_2 \beta_1 - \alpha_2 \alpha_1)}{(\beta_2 - \alpha_2)} \tag{6}$$

where $\alpha_1, \beta_1, \alpha_2, \beta_2$ are angles of two sides of L_{rn}, L_{sn} with respect to $Mid\ (L_{rn})$, $Mid\ (L_{sn})$ respectively. Two angles are computed due to lack of information about order of sides of reference diagonals being compared. The computation of θ_1, θ_2 before actual matching ensures less number of matches between $I_r(x,y), I_s(x,y)$. Initial transformation estimate, E_i, between reference convex hull of $I_r(x,y)$ and i[th] convex hull of $I_s(x,y)$ is

$$E_i = [Scale, \quad (x_t \quad y_t), \quad (\theta_1 \quad \theta_2)]$$

$$\forall\, i = \{1,2,\cdots,M\} \tag{7}$$

Having transformation parameters, Hausdorff distance is calculated as followings

$$H_i(R,S_i) = \max(h(R,S_i), \quad h(S_i,R))$$

$$R = R[I_r(x,y)] \; \forall \; i = \{1,2,\cdots,M\} \tag{8}$$

$$S_i = S_i[I_s(x,y)] \; \forall \; i = \{1,2,\cdots,N\}$$

$$where \; \mathrm{N} \neq \mathrm{M}$$

R represents reference convex hull from $I_r(x,y)$, S_i is i[th] convex hull from $I_s(x,y)$. Set of points used to compute Hausdorff distance between R, S_i consists of respective convex hull vertices. Number of computed convex hull in both images can be different due to quantization errors, occlusions, noise and measurement errors.

For fine level match and correction process, E_i is selected satisfying

$$\frac{AREA\ (R)}{AREA\ (R_c)} - \frac{AREA\ (S_i)}{AREA\ (S_{ic})} \leq T$$

$$AREA = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - y_i x_{i+1}) \tag{9}$$

where T, R_c, S_{ic} represents threshold, convex quadrilateral in $I_r(x,y)$ and i[th] convex quadrilateral in $I_s(x,y)$ respectively. Area of convex hull can be calculated using (9) where *n* shows number of vertices of convex hull. Equation (9) also holds for area calculation of non-convex hull regions. Ratio of areas between convex hull and its quadrilateral stays almost fixed for rigid transformations and this assumption is used to select pair of convex hull for further verification and investigation in image registration.

In fine level matching, original image data inside R, S_i is matched followed by correction process consisting of small adjustment in E_i to get local descent to minimize quantization errors. Multiple $E_i s$ satisfying (9) is possible due to varying number of vertices, size and orientation of convex hull.

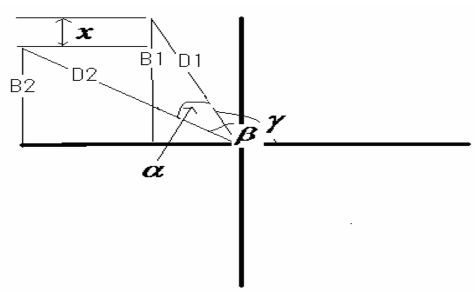


Figure. 2. Relationship between compared reference diagonals

The proposed method is computationally efficient with less number of fine level matches due to initial transformation estimate. At fine level matching, E_i is tested on original image data followed by verification process. A correct image registration should satisfy following relations between highest vertices of convex hulls in $I_r(x,y)$ and $I_s(x,y)$

$$\cot^{-1}\left[\cot(\alpha)-\frac{D_2}{D_1}[\operatorname{cosec}[\alpha]]\right]>\beta\ \forall\ \gamma=(\beta+\alpha)$$

similarly (10)

$$\cot^{-1}\left[\frac{D_2}{D_1}(\operatorname{cosec}(\alpha))-\cot(\alpha)\right]>\beta\ \forall\ \gamma=(\beta\text{-}\alpha)$$

$$x=D_1[\sin(\gamma)]-\sin(\beta).D_2\ \forall\ \gamma=(\beta-\alpha)$$

(11)

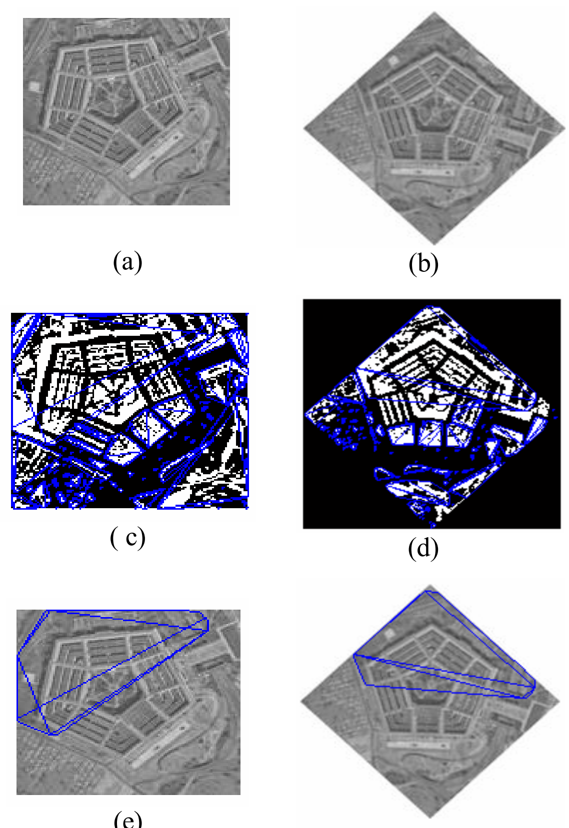
$$x=D_1[\sin(\gamma)]-\sin(\beta).D_2\ \forall\ \gamma=(\beta+\alpha)$$

where β, γ represent angles of highest vertices in S_i, R with respect to mid point of convex diagonal. α is the difference angle between highest vertices of R, S_i. D_1, D_2 represent distance between highest vertices and mid point of convex diagonals in R, S_i respectively as shown in figure 2. A correct image registration should produce difference of vertical distance, x, between reference point and highest vertices of R, S_i. Equations (10-11) give information about change in angle of highest convex hull vertex and vertical distance between highest vertex and reference points.

Only those estimated transformation parameters satisfying (10-11) are selected. Selection of transformation parameters based on minimum Hausdorff distance does not guarantee correct matching due to reasons: 1) different shapes can have same convex hull 2) Hausdorff distance is based not only on position but also on the set of points and relative distance 3) Hausdorff distance between nested convex hulls is less than distance between two same convex hulls with transformation difference.

IV. EXPERIMENTAL AND RESULTS

This part consists of experiment on registration of two images of a distant scene. The experiment is based on assumption of no occlusions for scene objects in $I_r(x,y), I_s(x,y)$. Figure 3-(a-b) show reference and sensed images i.e. $I_r(x,y)$, $I_s(x,y)$., (c-d) show images with computed convex hulls, reference diagonals, and convex quadrilaterals. Discrete coordinates cause disparity in image transformation. Applying Hausdorff distance directly on individual scene objects with all possible rotations can be computationally expensive.



(a) (b)

(c) (d)

(e) (f)

Figure. 3. Partial convex hull information for image registration

Reference convex hull in $I_r(x,y)$ is compared with convex hulls in $I_s(x,y)$ to compute $H_i(R,S_i)$ using (5-8). Pairs of R, $S_i\ \forall\, i=\{1,2,\ldots,N\}$ for fine level matching are selected based on criteria in (9). Figure 3-(e) shows reference convex hull from $I_r(x,y)$ to be compared with all convex hulls of $I_s(x,y)$. Figure 3-(f) shows matched convex hull in $I_s(x,y)$. Both images, used in experiment, are of same size with no occlusions or scene changes except rigid transformation. Candidate convex hull pairs $(R, S_i)\ \forall\ i \le N$ satisfy $H(R, S_i) \le T$ are matched using actual image data i.e. neighborhood edge map. Neighborhood edge map of convex hull is represented by circle, centered on reference point, of radius VX. Where $V > 0$ and X is equal to magnitude of reference diagonal.

The proposed method can also be extended to convex hull difference matching and using their relative geometrical information for transformation parameters estimate instead of comparing all convex hulls. This extension can also help in evaluating image registration parameters in case of occlusions not supported otherwise.

Table-1 shows number of convex hulls computed and matched, transformation parameters computed using (5-6).

TABLE. 1. ESTIMATED TRANSFORMATION PARAMETERS

	$I_r(x, y)$	$I_s(x, y)$
Total Convex Hulls	102	102
Image Size	128x128	128x128
Matched Convex Hull #	4	32
Rotation Parameter	-45^0	
Translation Parameter (x, y)	(47,27)	
Scaling	0.9928	

Table -1 shows estimated transformation parameters generated by the proposed method. Number of convex hull in sensed and reference images are equal with no occlusions in both images. While same number of convex hulls does not guarantee above assumption. Images used in experiment are both 128x128 size. Convex hulls are stored and sorted based on their size into two vectors – one for each image. Nested convex hulls at this stage are discarded as nested convex hulls produce smaller Hausdorff distance and lead to wrong transformation estimate. Sorted convex hulls in both vectors are compared using Hausdorff distance and only those convex hull pairs less than pre-defined threshold values, equation (9), are selected for fine level matching. Proposed method can also be used to produce input parameters for other image registration algorithms at fine level matching. Reference points from both convex hulls should overlap while computing Hausdorff distance so that Euclidean distance between respective convex hull pairs does not affect the actual dissimilarity measure.

Convex hull pair selection, based on Hausdorff distance, is followed by verification and correction process i.e. computed parameters should satisfy (10-11). Sometimes quantization errors, introduced due to inherent digital nature of images, can also affect correct image registration.

V. CONCLUSION

The proposed method deals with extraction of new, geometrically invariant feature set in images for fast registration. The proposed method is suitable for registration of distant scene images, where intensity information is effected due to varying imaging conditions, with good local accuracy. Usage of multiple objects geometric information can help to extend the proposed method for registration of symmetric object images. Extension of proposed method using match between convex hull difference and relative geometric information can also support registration of images with occluded objects. The proposed method can be used for real time input parameter estimation to other image registration algorithms requiring intensive computation for initial transformation estimation.

The proposed method can potentially be useful for remote sensing, environment monitoring and satellite image fusion for surveillance and guidance.

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