

# Implementation of Sensor Selection and Fusion Using Fuzzy Logic

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## Abstract

Different sensors may contain degrees of uncertainty and may be only reliable in particular situations, therefore sensor fusion and validation can be critical in complex redundant systems. This paper proposed a generic fuzzy logic algorithm for validation and fusion of uncertain sensor data. The system degrades marginal sensor data elegantly, while still removing obviously questionable data. Sensor data is represented as Gaussian curves. The sensor fusion problem is presented as determining a fused mean and standard deviation for the Gaussian of the output abstract sensor. Four variants of the fuzzy sensor fusion and validation system are presented and examined.

## 1. Introduction

Sensor fusion has long been of interest to designers of redundant sensor systems. Previous systems have primarily been based on Bayesian statistical models [1]. Some work has been done in fuzzy sensor fusion. We present a fuzzy sensor fusion system, which treats marginal sensor data in a fuzzy fashion during validation.

Sensor fusion technology has developed for many years and numerous researches had contributed to the evolution of the technology. The, Department of Defense (DoD) USA, has developed many prototype system for application in the areas of defense and surveillance. The data fusion group at the DoD Joint Directors of Laboratories (JDL) has developed numerous non-military applications in data fusion technology [2][3].

A process model based approach in sensor grading had been applied to the diagnosis of multiple sensor and actuator failures in automotive engines using a decision table relating all possible failure patterns to the residual code [4]. Another process model based approach used fuzzy identifiers to estimate the engine signals necessary to calculate the deviation from nominal internal combustion engine behavior is reported in [5]. A fuzzy statistical decision model in [6] used a single fuzzy membership to model a Gaussian sensor's mean and standard deviation.

Our system attempts to implement fuzzy sensor fusion and interpretation by abstracting the sensor model as a Gaussian distribution. We form fuzzy hypothesis about the validity and reliability of sensor data based on the standard deviation given to the sensor and the proximity of conflicting sensor readings using IF-AND-THEN rules. Fuzzy systems are particularly suitable for sensor fusion because quality judgements must be made at several levels. The intrinsic quality of a sensor reading must be established *a priori* by an expert. The relative quality of redundant sensor information must be assessed online. A new measure of data quality must be expressed for the final output.

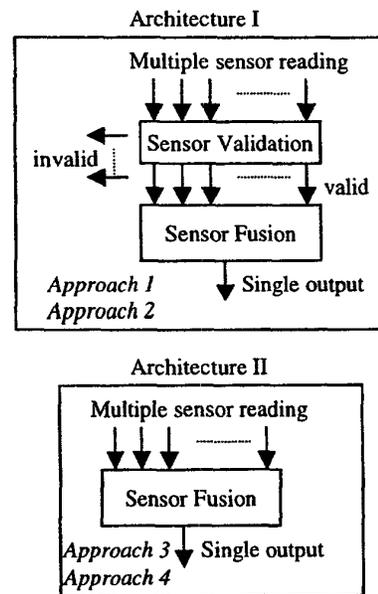


Figure 1. Sensor Validation and Fusion Diagram

## 2. Method

Sensors are modeled using Gaussian curves of equal area to denote sensor reliability. The mean of the curve is the current sensor reading. The standard deviation of the curve is the reliability of that particular sensor. By fixing the area of curves, less reliable sensor readings will receive a lower weighting during the center of mass fusion algorithm.

The four sensor validation and fusion approaches implemented are shown in Figure 1. In Architecture I, a crisp determination on sensor validity is made prior to fusion. In Architecture II, the sensors are assigned a weight based on a fuzzy fitness, and all sensors are included in the fusion step.

All systems employ the fuzzy sensor fusion engine. In Architecture I, the weights assigned to each sensor are crisp pass/fail decisions. In Architecture II, fuzzy weightings are assigned prior to fusion.

The standard deviation is calculated statistically formula in Approach 1. In Approach 2, a fuzzy linguistic model is used to perform a weighted fusion of sensor data based on subjective measures of data quality.

Architecture II employs fuzzy sensor validation. The use of fuzzy validation allows us to consider sensor readings, which are marginal without losing all sensor information. In approach 3, the weighting of mean and standard deviation is performed using the same rule base. In Approach 4, independent rule bases are used to assess the mean and standard deviation.

## 2. Fuzzy Engine

The fuzzy engine forms the core of our processing. It is used in sensor weighting, analysis, and fusion. The fuzzy engine is illustrated in Figure 2.

### 2.1.1 Fuzzification

Fuzzy sets are defined as trapezoidal or triangular membership functions. We chose trapezoidal and triangular membership functions because they have been shown to be a good compromise between effectiveness and efficiency. The formula for the trapezoidal and triangular membership function is

$$\mu_A(x; a_i, b_i, c_i, d_i) = \max(0, \min(\frac{x-a_i}{b_i-a_i}, 1, \frac{d_i-x}{d_i-c_i})) \quad (1)$$

where  $a_i, b_i, c_i, d_i$  are coordinates of the  $i$ -th trapezoid apexes for fuzzy set A.

Five membership functions ( $i=5$ ) are used in our algorithms, as shown in Fig. 3. The values of  $a_i, b_i, c_i, d_i$  are configurable by the user. When  $b = c$ , the membership function is triangular.

### 2.1.2 Rule Evaluation

Fuzzy IF-AND-THEN relationships can be used to reason about uncertain data. We have used a fuzzy IF-

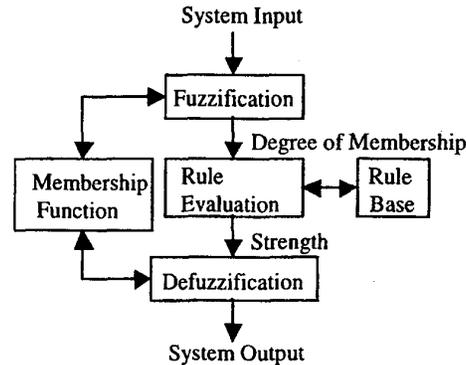


Figure 2. Fuzzy Engine Diagram

AND-THEN relationship to compare various aspects

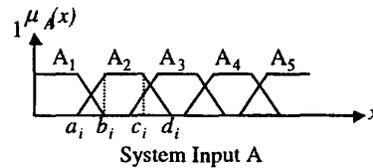


Figure 3. Fuzzy Set for System Input A

of the sensor fusion system such as sensor quality and appropriateness.

The minimum function is used to calculate the strengths of each rule. The maximum function is used to calculate the strength of all rules.

Rule 1: IF  $x_k$  is  $A_1$  and  $y_k$  is  $B_1$  THEN  $c_k$  is  $C_1$

Rule 2: IF  $x_k$  is  $A_2$  and  $y_k$  is  $B_2$  THEN  $c_k$  is  $C_2$

Strength of rule 1:

$$\mu_{C_1}(z_k) = \mu_{A_1}(x_k) \wedge \mu_{B_1}(y_k) \quad (2)$$

Strength of rule 2:

$$\mu_{C_2}(z_k) = \mu_{A_2}(x_k) \wedge \mu_{B_2}(y_k) \quad (3)$$

⋮

Strength of all rules:

$$\mu_{C}(z_k) = \mu_{C_1}(z_k) \vee \mu_{C_2}(z_k) \vee \dots \vee \mu_{C_N}(z_k) \quad (4)$$

### 2.1.3 Defuzzification

The strength of all rules from the rule evaluation process is defuzzified using the pre-defined membership functions the center of gravity method, as defined in (5) and shown in Fig. 4.

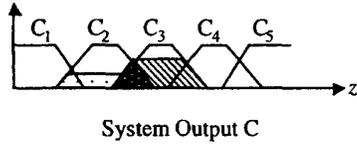


Figure 4. Fuzzy Set for System Output C

$$z_c = \frac{\int \mu_C(z_k) z_k dz_k}{\int \mu_C(z_k) dz_k}, 0 \leq z_c \leq 1 \quad (5)$$

### 3. Sensor Fusion and Validation

Our sensor fusion system is based on the assumption that the reliability of any measurement can be modeled as a Gaussian curve. By assuming that the sensor readings are Gaussian, we implement a generic system, which determines the fitness of the sensor without complex contextual reasoning. The sensors are modeled using (6).

$$f(X_i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(X_i - \mu)^2}{2\sigma^2}}, -3\sigma < X_i < +3\sigma \quad (6)$$

where  $\mu$  is the mean and  $\sigma$  is the standard defined as

$$\mu \equiv \frac{\sum_{i=1}^N X_i}{N} \quad (7)$$

$$\sigma \equiv \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N - 1}} \quad (8)$$

where  $X_i$  is the individual reading from single sensor and  $N$  is total number of readings.

#### 3.1 Method 1 and Method 2

The  $\mu_x$  (mean) and  $\sigma_x$  (standard deviation) are used as the two system inputs to the fuzzy engine in (1) for grading as follows

$$(x_k, y_k) = (\mu_x, \sigma_x) \quad (9)$$

and

$$z_k = \text{grade} \quad (10)$$

Each fuzzy set, A and B are labeled as  $\{A_1, A_2, A_3, A_4, A_5\} = \{\text{negative poor, negative accept, good, positive accept, positive poor}\}$  and  $\{B_1, B_2, B_3, B_4, B_5\} = \{\text{very small, small, medium, large, and very large}\}$  where *negative* in the A fuzzy set represents values below the good range and *positive* in the M fuzzy set represents values above the good range.

The 25 rules for method 1 are described in Table 1.

VS: very small, SL: small, MD: medium, LG: large, VL: very large NP: negative poor, NA: negative accept, GD: good, PA: positive accept, PP: positive poor VP: very poor, PR: poor, AC: accept, GD: good, VG: very good

Table 1. Knowledge Rule Base for Sensor Validation

$\sigma$	VS	SL	MD	LG	VL
$\mu$					
NP	VP	VP	VP	VP	VP
NA	VG	GD	AC	PR	VP
GD	VG	GD	AC	PR	VP
PA	VG	GD	AC	PR	VP
PP	VP	VP	VP	VP	VP

The fuzzy set for C (grade) is labeled  $\{C_1, C_2, C_3, C_4, C_5\} = \{\text{very poor, poor, accept, good, very good}\}$ . The grade of the sensor varies between 0 and 1.

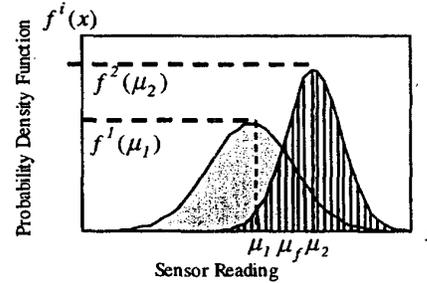


Figure 5. Fused mean calculation

A pre-defined threshold determines the accept/reject decision. We use 0.5 as the threshold value in our approach, which is the center value of  $C_3$  (Accept) fuzzy set.

$$\text{grade} = \begin{cases} \text{Accept}, Z_c \geq \text{threshold} \\ \text{Reject}, Z_c < \text{threshold} \end{cases} \quad (11)$$

Sensors with  $Z_c$  greater than the threshold are fused. The sensor reading (*mean*) for each distribution is the centroid of its membership function. Therefore the fused mean can be obtained using the center-of-gravity method formulated in (12) and illustrated in Figure 5.

$$\mu_{\text{fused}} = \frac{\int f^i(x_k) x_k dx_k}{\int f^i(x_k) dx_k} \quad (12)$$

Because each Gaussian curve has the same area, the equation can be re-formulated as

$$\mu_{\text{fused}} = \frac{\text{summation of the sensor reading}}{\text{Number of sensor}} \quad (13)$$

Ru-Jen Chao *et al.* [7] derived the same result for the expected value of a fuzzy-random variable based on a fuzzy-random probability function.

Subsequently we must calculate the fused variance to new mean obtained in the previous section. Method 1 is uses a statistical method. All sensors' Gaussian curves are weighted and summed as follows

$$\sigma_{fused} \equiv \sqrt{\frac{\sum_{i=1}^N \sum_{k=1}^M (x_k^i - \mu_{fused})^2 \times f^i(x)}{N \times M - 1}} \quad (14)$$

where  $N$  is the number of sensor number,  $M$  is the number of data points, and  $f^i(x)$  is the probability value. We used 500 data points to represent the curve.

In method 2, the standard deviation, and the distance between each mean and the new mean,  $|\mu_k - \mu_{fused}|$ , are inputs to the fuzzy engine, illustrated in Figure 6.

$$(x_k, y_k) = (\sigma_k, |\mu_k - \mu_{fused}|) \quad (15)$$

and

$$z_k = \text{weight} \quad (16)$$

Each fuzzy set, A (standard deviation) and B (Distance) is labeled as  $\{A_1, A_2, A_3, A_4, A_5\} = \{\text{very small, small, medium, large, and very large}\}$  and  $\{B_1, B_2, B_3, B_4, B_5\} = \{\text{very close, close, medium, far, very far}\}$

The 25 rules for method 2 are shown in Table 2.

VS: very small, SL: small, MD: medium, LG: large, VL: very large VC: very close, CL: close, MD: medium, FR: far, VF: very far VL: very low, LW: low, MD: medium, HI: high, VH: very high

Table 2. Knowledge Rule Base for Method 2

$\sigma$	VS	SL	MD	LG	VL
D	VS	SL	MD	LG	VL
VC	VH	VH	VH	LW	VL
CL	HI	HI	HI	LW	VL
MD	MD	MD	MD	LW	VL
FR	LW	LW	LW	LW	VL
VF	VL	VL	VL	LW	VL

The fuzzy set for  $C$  (weight) is labeled as  $\{C_1, C_2, C_3, C_4, C_5\} = \{\text{very low, low, medium, high, very high}\}$

The strength of all rules from the rule evaluation process is defuzzified using the center of gravity method. The fused standard deviation can be obtained by the weighted average of each sensor's standard deviation as

$$\sigma_{fused} = \frac{\int \sigma_i z_C^i}{\int z_C^i} \quad (17)$$

### 2.2.2 Method 3 and Method 4

In methods 1 and 2, the sensor fusion was performed after marginal sensors were eliminated. In methods 3 and 4, the fusion is a weighted sum based on a fuzzy sensor fitness calculation that varies between 0 (unfit)

and 1 (perfect). The fuzzy interpretation of fitness more accurately captures information contained in marginal sensor readings.

Method 3 uses the same fitness calculation and table as method 1, but the results are used as weights instead of crisp measures. The calculation of the mean and standard deviation becomes

$$\mu_{fused} = \frac{\int \mu_i z_C^i}{\int z_C^i} \quad (18)$$

and

$$\sigma_{fused} = \frac{\int \sigma_i z_C^i}{\int z_C^i} \quad (19)$$

Method 4 separates the fuzzy inference process. The mean and standard deviation weightings are determined using separate rule bases, allowing different weights to be assigned to each.

The fuzzy set, A (standard deviation) is labeled as  $\{A_1, A_2, A_3, A_4, A_5\} = \{\text{very small, small, medium, large, and very large}\}$

Five rules are applied with one condition in the if statement as follows

Table 3: Knowledge Rule Base for Standard Deviation

$\sigma$	VS	SL	MD	HI	VH
	VH	HI	MD	LW	VL

Triangular membership functions are used with the centers at [0.167, 0.333, 0.5, 0.667, 0.833]. The fuzzy set for  $C$  (weight) is labeled as  $\{C_1, C_2, C_3, C_4, C_5\} = \{\text{very low, low, medium, high, very high}\}$ .

The fuzzy set, A (mean) is labeled as  $\{A_1, A_2, A_3, A_4, A_5\} = \{\text{negative poor, negative accept, good, positive accept, positive poor}\}$ . Five rules are applied with one condition in the if statement as follows

Table 4: Knowledge Rule Base for Mean

$\mu$	NP or PP	NA or PA	GD
	LW	MD	HI

Triangular membership functions are used, centered at [0.25, 0.5, 0.75].

The fuzzy set for  $C$  (weight) is labeled as  $\{C_1, C_2, C_3, C_4, C_5\} = \{\text{low, medium, high}\}$

The new standard deviation is calculated as

$$\mu_{fused} = \frac{\int \mu_i z_R^i}{\int z_R^i} \quad (20)$$

#### 4. Result

Eight sensors were simulated with random variables normally distributed with  $\mu$  (mean) between 15 and 20, and  $\sigma$  (standard deviation) between 0.6 and 3. We manually changed some parameters to unacceptable values to demonstrate the validation algorithm. The data range for Gaussian distribution is set to between  $\mu - 3\sigma$  and  $\mu + 3\sigma$  which will contains 99.7% of the curve. The eight nominal sensors' means and standard deviations are listed in Table 3. Figure 6 shows the eight Gaussian approximation curves using the data in Table 3 and (4).

Table 5: Sensor reading distribution parameters

	$\mu$	$\sigma$
ID 1	36.32	1.272
ID 2	59.15	2.58
ID 3	1.95	2.9688
ID 4	-10.555	1.1448
ID 5	18.48	5.9544
ID 6	16.22	11.8816
ID 7	15.535	2.1
ID 8	20.385	2.6384

Table 6 shows the result of validation for the 8 nominal sensors, the grade is the output from the fuzzy inference engine and used as the accept/reject criteria. The threshold is 0.5, rejecting sensors 1 to 6.

Table 6: Sensor Validation Result

Sensor	Mean	Standard deviation	Grade	Decision
ID 1	36.32	1.272	0.1365	Reject
ID 2	59.15	2.58	0	Reject
ID 3	1.95	2.9688	0.1365	Reject
ID 4	-10.555	1.1448	0	Reject
ID 5	18.48	5.9544	0.3185	Reject
ID 6	16.22	11.8816	0	Reject
ID 7	15.535	2.1	0.7656132	Accept
ID 8	20.385	2.6384	0.6815	Accept

Table 7 shows the result of method 2 for the eight nominal sensors. The grades from table 6 were used as the sensor weights standard deviation fusion. The validation process had rejected sensors 1 to 6. Therefore only weights of sensors 7 and 8 are included.

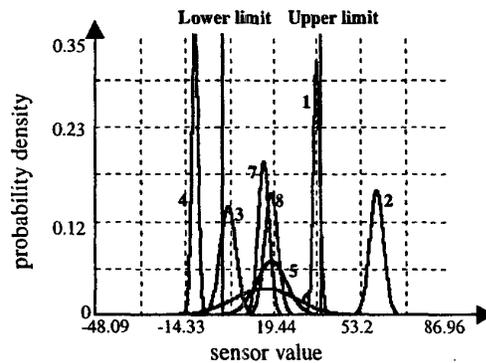


Figure 6. Distribution of 8 sensors

Table 7: Sensor Rating Result (Method 2)

Sensor	$\sigma$	$D$	Weight
ID 1	1.272	18.36	0
ID 2	2.58	41.19	0
ID 3	2.9688	16.01	0
ID 4	1.1448	28.515	0
ID 5	5.9544	0.5200005	0.3185
ID 6	11.8816	1.74	0
ID 7	2.1	2.424999	0.692553
ID 8	2.6384	2.425001	0.6881664

The weight calculations for method 3 are not shown because they are identical to the grade calculations for methods 1 and 2. Instead of using the results as a grade for sensor validation, they were taken as weighting factors for mean and standard deviation. In this case, partial information from sensors 1, 3 and 5 were added.

Table 8 shows the method 4 results for the eight nominal sensors. The independently calculated weights for the standard deviation and mean were used to calculate the fused mean and standard deviation using the center-of-mass approach.

Table 8. Sensor Rating Result (method 4)

Sensor	$\mu$	Weight ( $\mu$ )	$\sigma$	Weight ( $\sigma$ )
ID 1	36.32	0.25	1.272	0.833
ID 2	59.15	0	2.58	0.667
ID 3	1.95	0.25	2.9688	0.667
ID 4	-10.555	0	1.1448	0.833
ID 5	18.48	0.75	5.9544	0.333
ID 6	16.22	0.674	11.8816	0
ID 7	15.535	0.624	2.1	0.739
ID 8	20.385	0.75	2.6384	0.667

Table 9 shows the fusion result of all four methods.

Table 9. Sensor Fusion Result with four methods

Fusion Result	$\mu$	$\sigma$
Method 1	17.96	3.381882
Method 2	17.96	2.368345
Method 3	16.95034	3.00869
Method 4	17.99344477	2.32303304

It is apparent from Table 9 that method 4 provides the smallest standard distribution in this particular case. We feel that more accurate results can be obtained by using weighted averages during sensor validation.

## 5. Conclusion

In this paper, we have proposed a generic knowledge rule based architecture that provides the capability for validation and fusion of multiple sensor readings. The fuzzy approach for fused mean and standard deviation combined with the fuzzy validation process is favorable compared with others because it preserves marginal sensor information.

Our future work will focus on expanding the capabilities of the engine and sensor modeling. While we wished the fusion engine to be model free, there is valuable data in sensor models. Allowing the standard deviation (reliability) to change depending on a model of sensor behavior would increase the utility of the system. We will test the system using distributed gas and temperature sensors to determine its functionality in an industrial environment.

## 6. References

1. Richard R. Brooks and S. S. Iyengar, *Multi-Sensor Fusion: Fundamentals and Applications with Software*, Prentice Hall, Toronto, 1998.
2. D. L. Hall, R.J. Linn, and J. Linas, "A Survey of Data Fusion Systems," *Proceedings of the SPIE Conference on Data Structures and Target Classification*, pp. 13-29, April 1991.
3. J.A. Stover, D.L. Hall, R.E. Gibson, "A fuzzy-Logic Architecture for Autonomous Multisensor Data Fusion," *IEEE Transactions on Industrial Electronics*, Vol. 43, Issue 3, pp. 403 - 410
4. P. Hsu, K. Lin, and L. Shen, "Diagnosis of Multiple Sensor and Actuator Failures in Automotive Engines," *IEEE Transaction on Vehicular Technology*, Vol. 44, No. 5, pp. 779-789, November 1995.
5. E.G.Laukonen, K.M. Passino, V. Krishnaswami, G.-C. Luh, G. Rizzoni, "Fault Detection and Isolation for an Experimental Internal Combustion Engine via Fuzzy Identification," *IEEE Transactions on Control Systems Technology*, Vol. 3, No. 3, pp. 347-355, September 1995.
6. M. Rokouzzaman, R.G. Gosine, "Adaptive fuzzy-statistical decision model to grade sensor data," *IEEE 1997 Canadian Conference on Electrical and Computer Engineering, 1997. Engineering Innovation: Voyage of Discovery*, Vol. 2, pp 773-776, May 1997.
7. Ru-Jen Chao; Ayyub, B.M., "Distributions with Fuzziness and Randomness," *Third International Symposium on Uncertainty Modeling and Analysis, 1995, and Annual Conference of the North American Fuzzy Information Processing Society. Proceedings of ISUMA - NAFIPS '95*, pp. 668 - 673, Sept. 1995.