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MODEL IDENTIFICATION FOR FUZZY DYNAMIC SYSTEMS

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Abstract

This paper presents methods for identifying the structure of a fuzzy model and determination of a fuzzy ruleset which maps input data to output data of a dynamic system. The structure identification is based on the evaluation of 'conflicting rules' in fuzzy associative memory (FAM) cells, and the primary ruleset is obtained by using the adaptive fuzzy associative memory (AFAM) method. A neural-network-based error compensator is developed to improve the accuracy of the fuzzy model.

1 Introduction

Based on the concept of "referential fuzzy sets", a discrete-time fuzzy relational model for a single-input single-output (SISO) system may be written as [1]

$$y(t) = y(t-\tau_y)\circ\ldots\circ y(t-n_y\tau_y)\circ u(t-\tau_u)\circ\ldots\circ u(t-n_u\tau_u)\circ R \quad (1)$$

where $y(\cdot)$ is the system output, $u(\cdot)$ is the system input and R is the relation between the inputs and the outputs, which will enable the response computation. The symbol "o" denotes the fuzzy composition operator. Note that τ_y and τ_u of are the sampling times (delays) of the output and input and n_y and n_u represent the denominator order and the numerator order, respectively, of the discrete, input-output system. The fuzzy model identification considered in this paper includes fuzzy order identification and fuzzy relation identification.

2 Fuzzy Order Identification

The method proposed for order identification is based on the idea that the number of conflicting FAM rules should be very small if the parameters used to generate the FAM rules are correct. FAM rules can be generated using the DCL-AVQ algorithm [2]. Synaptic vectors are distributed into FAM cells by the adaptive FAM rule generation procedure. Idealy, in each FAM cell, for a given input fuzzy set, the output fuzzy set should be unique. However, very often different fuzzy sets occurs in one FAM cell. When this situation takes place, we say that there are conflicting FAM rules.

Conceptually it is clear that fewer the number of such conflicting rules, better the model. This can also be explained by examining the following equation which has often been used to validate a fuzzy model:

$$P = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}(i) - y(i))^2. \quad (2)$$

Here P represents an accuracy measurement for a given fuzzy model, N is the number of sampling points, $\hat{y}(i)$ is the defuzzified model output at the i^{th} data point, and $y(i)$ is the i^{th} nonfuzzy observation. A smaller P value represents a more accurate model.

In the process of adaptive FAM rule generation, when sample points with more than one fuzzy set appear in a FAM cell, the consequent fuzzy sets in each cell are summed and the centroidal defuzzification is employed. The consequent fuzzy set for each FAM rule is that output fuzzy-set value with the centroid nearest to the defuzzified centroid of the corresponding cell. It is obvious that a greater error is introduced in this process to those sample points which had different fuzzy sets from the fuzzy set assigned to the FAM cell, and consequently larger P is resulted.

Therefore the issue of fuzzy order identification becomes a problem of looking for delays τ_y and τ_u and the system order parameters n_y and n_u at which the number of conflicting FAM rules is a minimum. In other words, we are looking for values of τ_y, τ_u, n_y and n_u with which the most compact peaks can be generated in the FAM cells. The detailed implementation of this procedure is described in Section 4, through a numerical example.

3 Fuzzy Relation Identification

The key issue in fuzzy modeling is to derive a primary fuzzy ruleset which can be used to map input data to output data. This gives the "relation" of the fuzzy model. We will first generate such a ruleset by directly using the AFAM method, and then propose a method to improve the accuracy of the fuzzy model.

Primary Ruleset Generation

The generation of the primary ruleset can be accomplished in the following three stages:

1. Determine the quantization level and estimate the membership functions.
2. Given the input-output sample vectors, use the DCL-AVQ algorithm to find the pattern-class centroids.
3. Partition the product space into FAM cells. Sum up the fuzzy sets in each cell and employ the centroidal defuzzification which is given by the following equation:

$$\bar{x} = \frac{\sum_{x\in X} x\mu_A(x)}{\sum_{x\in X}\mu_A(x)} \quad (3)$$

where $\bar{x}$ is a crisp scalar value, X is a finite set with generic element x and $\mu_A(x)$ is the membership grade of A at x.

Once the primary ruleset is obtained, the system is considered to be linguistically identified. For a given set of input data, how to predict the output using the fuzzy model is addressed next.

Output Prediction and Accuracy Analysis

Assuming that a fuzzy-set pair (A, B) is associated with the input-output pair and M is a fuzzy associative memory matrix, fuzzy-set B can be obtained by the compositional rule of inference; thus

$$B = A\circ M, \quad (4)$$

Specifically, through sup-min composition, the membership function of B can be obtained by

$$\mu_B(y) = \sup_{y\in Y}\min\left(\mu_A(x), \mu_M(x,y)\right) \quad (5)$$

where μ_B represents the membership function of the output variable, μ_A is the membership function of the input variable, and μ_M is the membership function of M. The membership function of M is formed by the constituent membership functions, using the "min" operation for implication. This method of obtaining M is analogous to model identification in conventional hard control. The inference process of obtaining B is analogous to the output prediction using the identified model. If certain conditions are satisfied, a perfect recall can be achieved.

However, since the fuzzy model uses discretised variables, the accuracy that can be achieved is usually low with a given set of membership functions. This paper proposes to use a neural-network (NN) based error compensator to improve the model accuracy. The basic principle is briefly described here. In the training phase, the crisp input signal is fed into the fuzzy model which generates the predicted fuzzy output. The input signal $u(t)$, the predicted fuzzy output $\hat{y}$ and the reference output $y_d(t)$ are then fed into the fuzzy NN for the training phase. Once the NN is trained, a group of weights are derived, which can be subsequently used in the on-line operation phase. In this phase, input is fed into the fuzzy model and a crisp output is generated by the fuzzy NN.

4 Experimental Results

The gas furnace data of Box and Jenkins have often been used as a standard for the validation of identification techniques. As described in Section 2, the technique used to identify the system order is based on the evaluation of conflicting FAM rules. In this experiment, FAM cells are defined as follows. The input gas flow rate $-2.716 <= u(t) <= 2.834$ is divided into five uniform intervals [-2.716, -1.606), [-1.606, -0.496), $[-0.496, 0.614)$, [0.614, 1.724), [1.724, 2.834], which respectively represent NH (negative high), NL (negative low), Z (zero), PL (positive low) and PH (positive high). Similarly, the output CO_2 concentration $y(t)$ is partitioned into five uniform intervals [45.6, 48.58), [48.58, 51.56), [51.56, 54.54), [54.54, 57.52), [57.52, 60.5], which are labeled as VS (very small), SM (small), MD (medium), LG (large) and VL (very large).

To evaluate the conflicting FAM rules, two procedures have been developed, which are referred to as the ***Sharpness Method*** and the ***Signal-to-Noise Ratio Method***.

Sharpness Method: Suppose that XI represents the interval which has the largest number of rules. The sharpness of a FAM cell is defined as

$$sp = a_1(R_p - R_n) + a_2(R_p - \alpha R_{nn}) \quad (6)$$

where sp represents the sharpness of a FAM cell, R_p is the number of rules in XI, R_n is the number of rules in the immediate neighborhood of XI, R_{nn} the next neighborhood of XI, and a_1, a_2, and ***alpha*** denote scaling factors. The sum (denoted by SP) of sp in all FAM cells represents a measurement of the degree of conflict between rules. It is clear that a larger SP represents a smaller conflict between rules.

Figure 1(a) graphically shows the distribution of the SP values. Each curve represents the variation of the SP values with respect to the normalised input delay τ_u, for a specific value for normalised τ_y, the top curve being for $\tau_y = 1$. It can be easily seen that the maximum SP is achieved when $\tau_u = 5$ and $\tau_y = 1$.

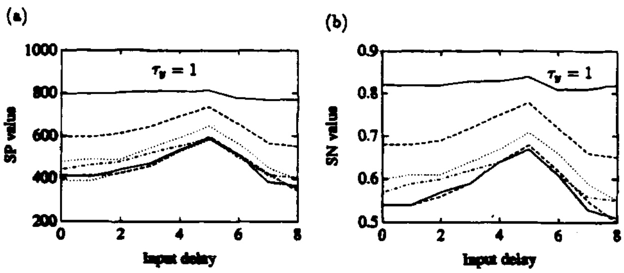

Figure 1: Graphs of the variation of SP and SN for different τ_u and τ_y. a) Sharpness method. b) Signal-to-noise ratio method.

"Signal-to-Noise" Ratio Method: The Sharpness Method is applicable when the order parameters n_u and n_y of the system are known. The Signal-to-Noise Ratio Method is applicable even when the order parameters are not known. The Signal-to-Noise Ratio (SN) is defined as

$$SN = \sum_{i=1}^{N_f} \frac{R_{p_i}}{R_{c_i} - R_{p_i}} \quad (7)$$

where N_f is the number of FAM cells, R_{c_i} the number of rules in the i^{th} FAM cell and R_{p_i} the number of rules in the XI of the i^{th} FAM cell. Figure 1(b) graphically shows the SN ratio computed for the gas furnace data with the normalised τ_u ranging from 0 to 8 and normalised τ_y ranging from 1 to 6. It is clear that the structure identified here is the same as in the Sharpness Method; i.e., $\tau_u = 5$ and $\tau_y = 1$.

Having identified the system order of the gas furnace data in this manner, we can conclude that the rules have the following structure:

$$(y(t-1), u(t-5)) \Rightarrow y(t).$$

To generate the primary ruleset, triangle membership functions are used for the fuzzy sets of the input and output variables, respectively. By using AFAM, the primary ruleset consisting of 21 rules is obtained. Compositional rule of inference and the centroidal defuzzification method are then applied to generate the predicted output. Figure 2(a) shows the desired output (denoted by yd in the figure and shown as a solid line) and the predicted output $\hat{y}$ (denoted by yr in the figure and shown as a broken line) by the fuzzy model. To improve the accuracy of the fuzzy model, a neural-network-based error compensator is then incorporated. Figure 2(b) shows the desired output (in solid line) and the output by the fuzzy model with the NN-compensator (denoted by ynn and shown in broken line). By using the method of fuzzy model validation, as expressed by equation (2), the fuzzy model with an NN-compensator can provide an accuracy of $P = 0.1$.

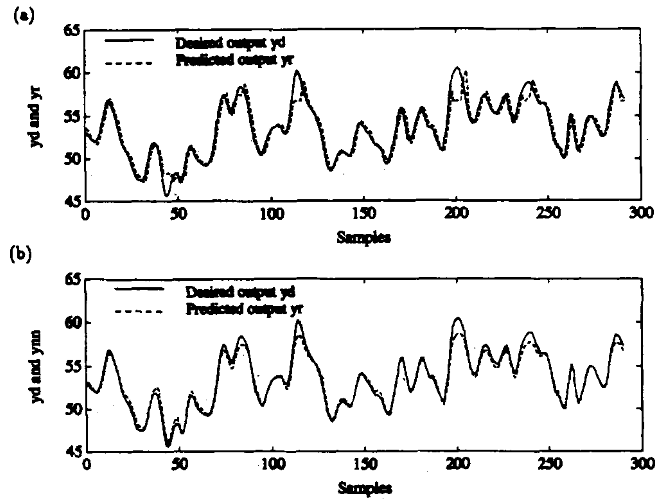

Figure 2: The desired output and the output predicted by the fuzzy model. a) Without the NN compensator. b) With the NN compensator.

5 Conclusions

A fuzzy model may be applied to control and tune complex systems including incompletely known, and nonlinear systems with time-varying characteristics. Such a model should possess adaptive and self-learning characteristics in general. Neural networks are applicable in this context. In this paper, we have explored the use of AFAM for the model identification of a fuzzy system. A method to identify the system structure based on the evaluation of conflicting FAM rules, was presented. After identifying the system order (structure), a primary ruleset can be generated using the AFAM method. Use of the fuzzy model to predict the system output was described, and a method was presented to improve the model accuracy. Experimental data were used to validate the methods. The results showed that the methods can be useful in practical applications. Development of special neural network architectures that can tune the membership functions optimally and at the same time take care of the rule changes, will be useful. The behavior of such neural networks can be easily interpreted in the form of fuzzy if-then rules.

References

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2 Kosko, B., *Neural Networks and Fuzzy Systems*, Prentice Hall, Englewood Cliffs, NJ, 1992.